# Introduction to QCD

Lecture 2

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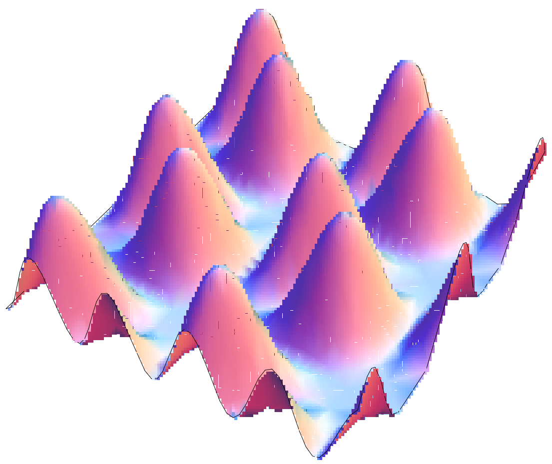
HUGS 2016

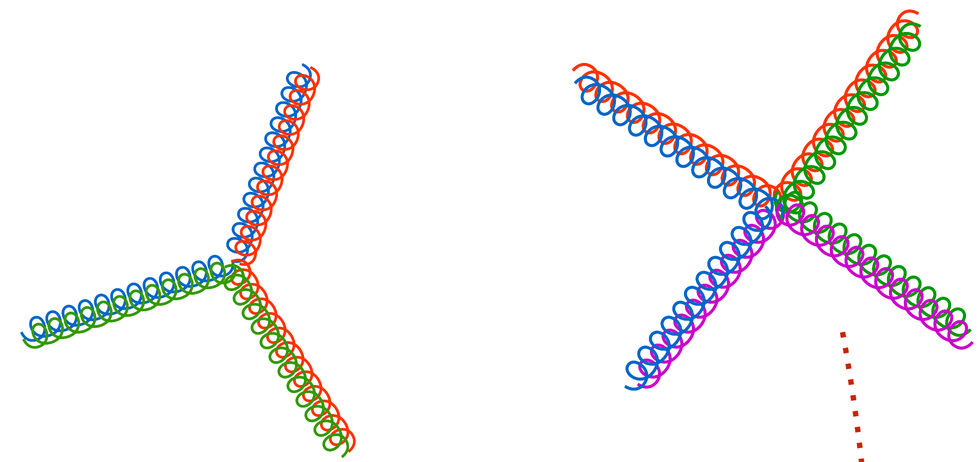
Jefferson Lab

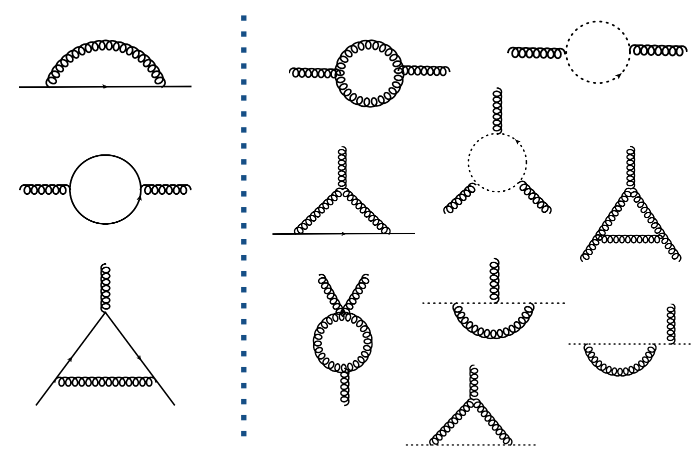
# What do we know now

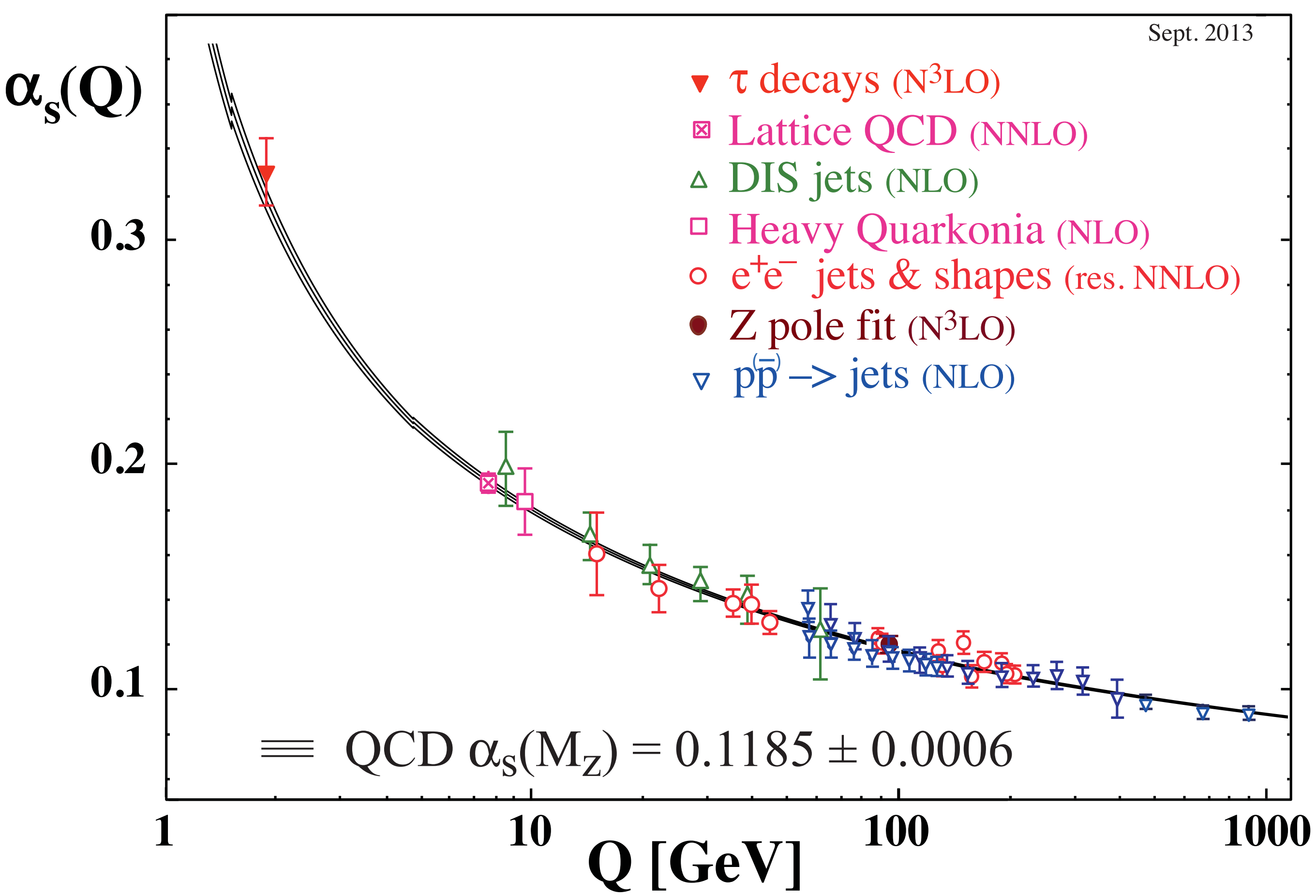
$$\psi(x) \to e^{i\alpha^a(x)t^a}\psi(x)$$

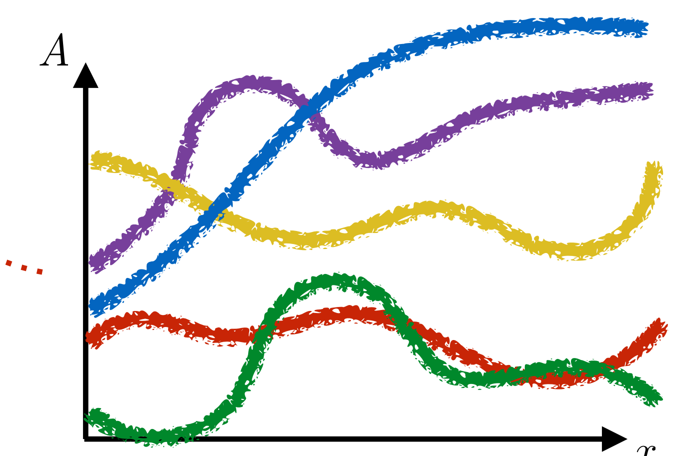
$$\mathcal{L}_{QCD} = \bar{\psi}(i\partial\!\!\!/ - m)\psi - \frac{1}{4}F^{a2}_{\mu\nu} + g\bar{\psi}\gamma^\mu t^a \psi A^a_\mu$$





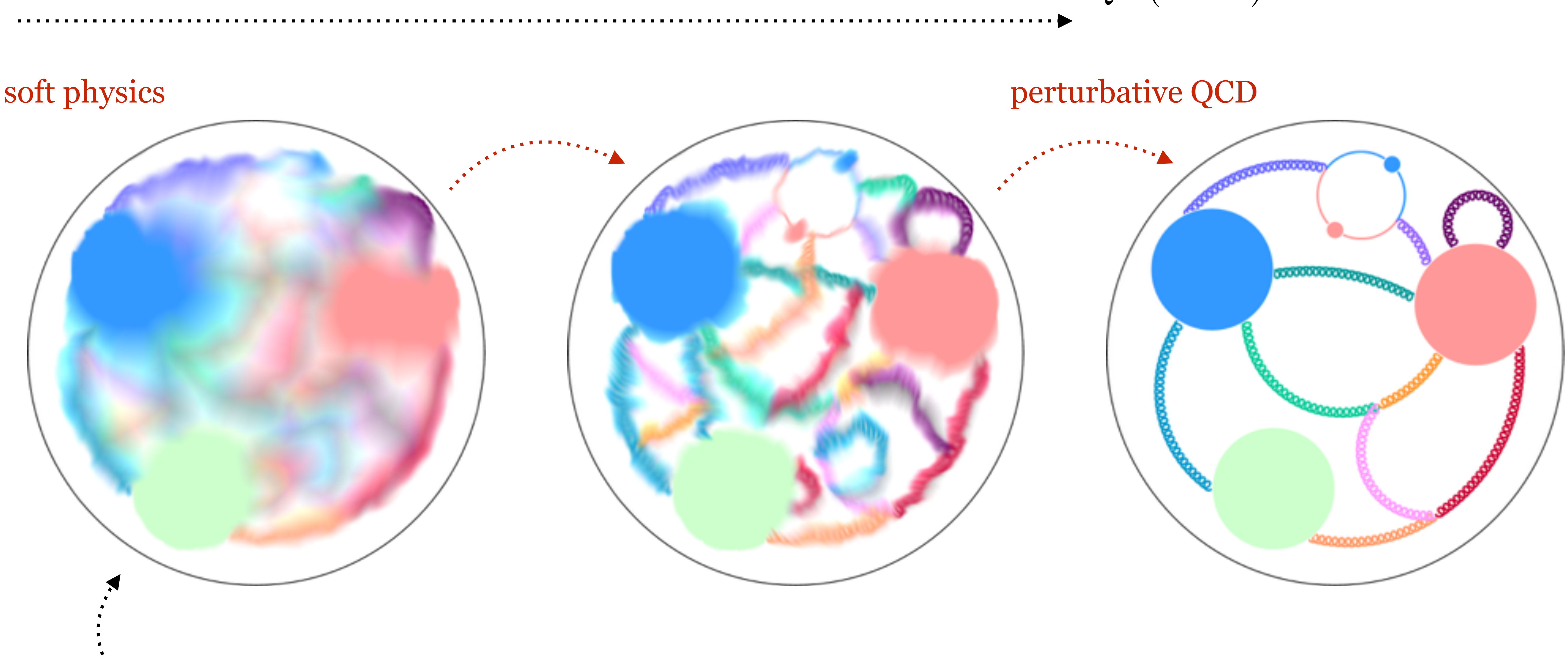




# QCD: Hadron at different scales

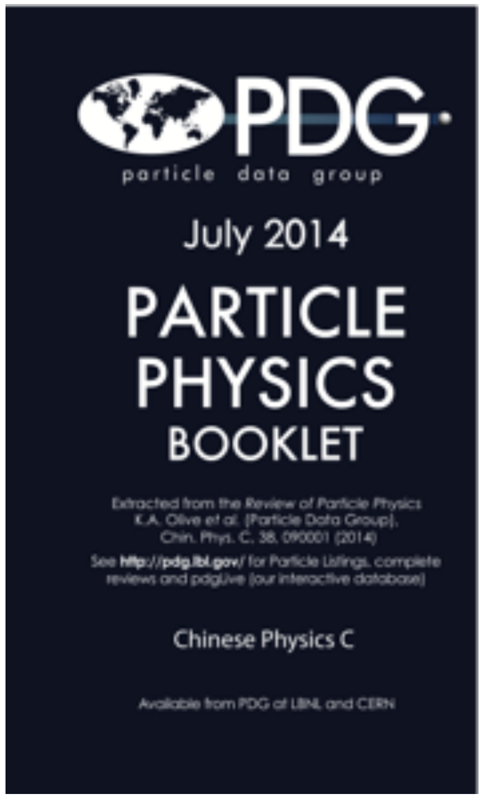
$Q^2 \ (GeV^2)$




Quark and gluons don't exist as asymptotic states

Where does $Q^2$ come from?

# Hadron radius




150 Baryon Summary Table

## N BARYONS
## $(S = 0, I = 1/2)$

$p, N^+ = uud$; $n, N^0 = udd$

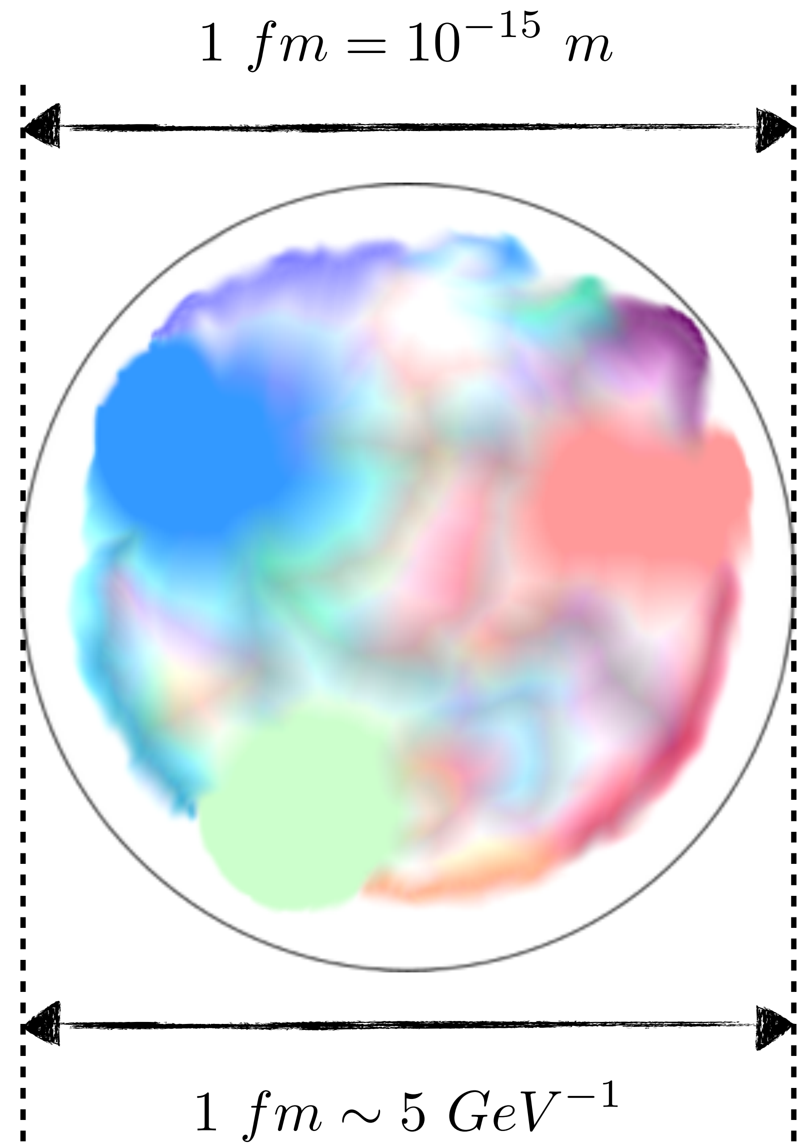
**p** $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$

Mass $m = 1.00727646681 \pm 0.00000000009$ u
Mass $m = 938.272046 \pm 0.000021$ MeV [a]
$|m_p - m_{\overline{p}}|/m_p < 7 \times 10^{-10}$, CL = 90% [b]
$|\frac{q_{\overline{p}}}{m_{\overline{p}}}|/(\frac{q_p}{m_p}) = 0.99999999991 \pm 0.00000000009$
$|q_p + q_{\overline{p}}|/e < 7 \times 10^{-10}$, CL = 90% [b]
$|q_p + q_e|/e < 1 \times 10^{-21}$ [c]
Magnetic moment $\mu = 2.792847356 \pm 0.000000023\ \mu_N$
$(\mu_p + \mu_{\overline{p}}) / \mu_p = (0 \pm 5) \times 10^{-6}$
Electric dipole moment $d < 0.54 \times 10^{-23}$ $e$cm
Electric polarizability $\alpha = (11.2 \pm 0.4) \times 10^{-4}$ fm$^3$
Magnetic polarizability $\beta = (2.5 \pm 0.4) \times 10^{-4}$ fm$^3$ (S = 1.2)
Charge radius, $\mu p$ Lamb shift $= 0.84087 \pm 0.00039$ fm [d]
Charge radius, $ep$ CODATA value $= 0.8775 \pm 0.0051$ fm [d]
Magnetic radius $= 0.777 \pm 0.016$ fm
Mean life $\tau > 2.1 \times 10^{29}$ years, CL = 90% [e] ($p \to$ invisible mode)
Mean life $\tau > 10^{31}$ to $10^{33}$ years [e] (mode dependent)

See the "Note on Nucleon Decay" in our 1994 edition (Phys. Rev. **D50**, 1173) for a short review.

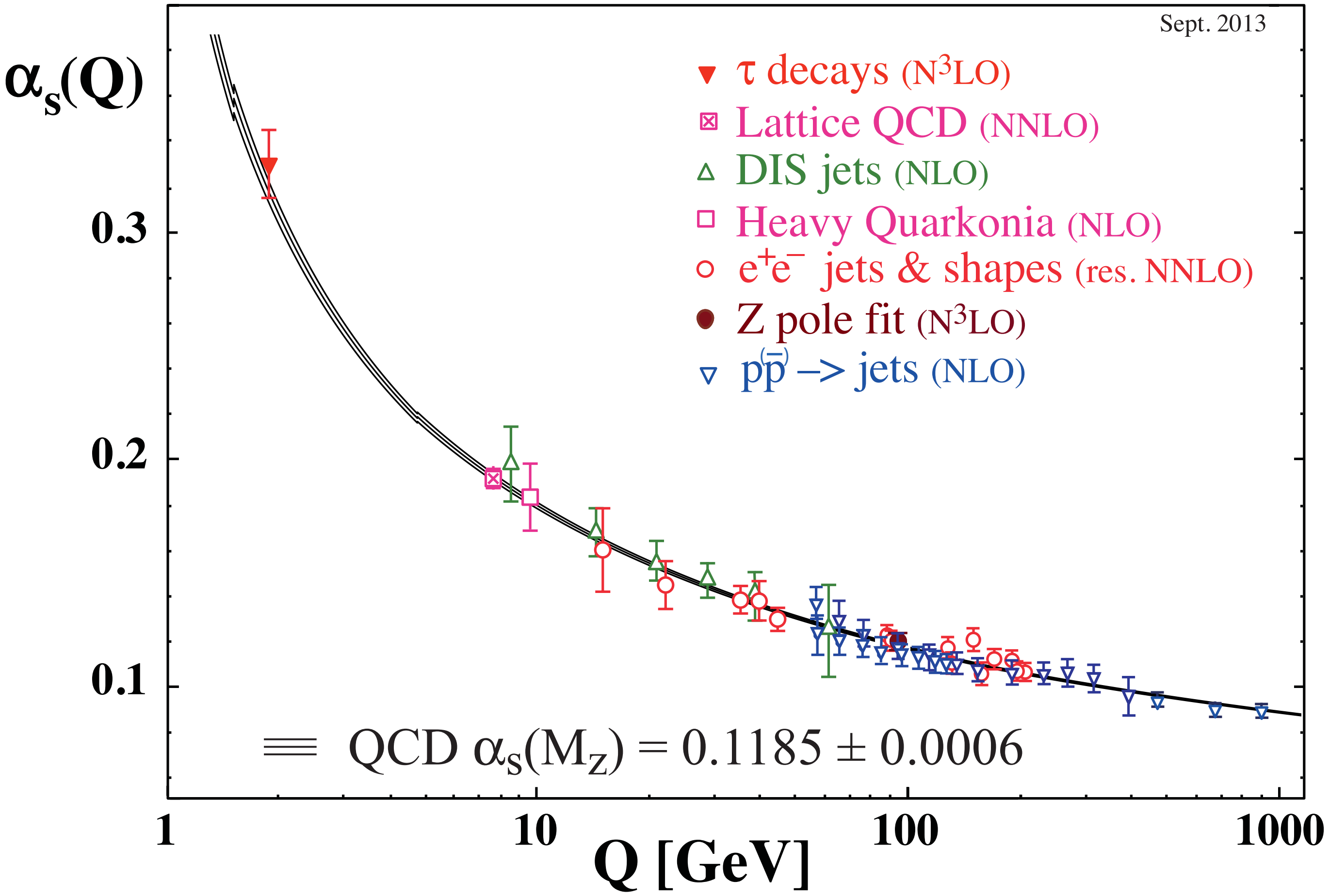
The "partial mean life" limits tabulated here are the limits on $\tau/B_i$, where $\tau$ is the total mean life and $B_i$ is the branching fraction for the mode in question. For $N$ decays, $p$ and $n$ indicate proton and neutron partial lifetimes.

| **p DECAY MODES** | Partial mean life ($10^{30}$ years) | Confidence level | $p$ (MeV/$c$) |
|---|---|---|---|
| **Antilepton + meson** | | | |
| $N \to e^+\pi$ | > 2000 (n), > 8200 (p) | 90% | 459 |
| $N \to \mu^+\pi$ | > 1000 (n), > 6600 (p) | 90% | 453 |
| $N \to \nu\pi$ | > 112 (n), > 16 (p) | 90% | 459 |
| $p \to e^+\eta$ | > 4200 | 90% | 309 |
| $p \to \mu^+\eta$ | > 1300 | 90% | 297 |
| $n \to \nu\eta$ | > 158 | 90% | 310 |
| $N \to e^+\rho$ | > 217 (n), > 710 (p) | 90% | 149 |
| $N \to \mu^+\rho$ | > 228 (n), > 160 (p) | 90% | 113 |
| $N \to \nu\rho$ | > 19 (n), > 162 (p) | 90% | 149 |
| $p \to e^+\omega$ | > 320 | 90% | 143 |
| $p \to \mu^+\omega$ | > 780 | 90% | 105 |
| $n \to \nu\omega$ | > 108 | 90% | 144 |
| $N \to e^+K$ | > 17 (n), > 1000 (p) | 90% | 339 |
| $N \to \mu^+K$ | > 26 (n), > 1600 (p) | 90% | 329 |
| $N \to \nu K$ | > 86 (n), > 2300 (p) | 90% | 339 |
| $n \to \nu K^0_S$ | > 260 | 90% | 338 |
| $p \to e^+K^*(892)^0$ | > 84 | 90% | 45 |
| $N \to \nu K^*(892)$ | > 78 (n), > 51 (p) | 90% | 45 |

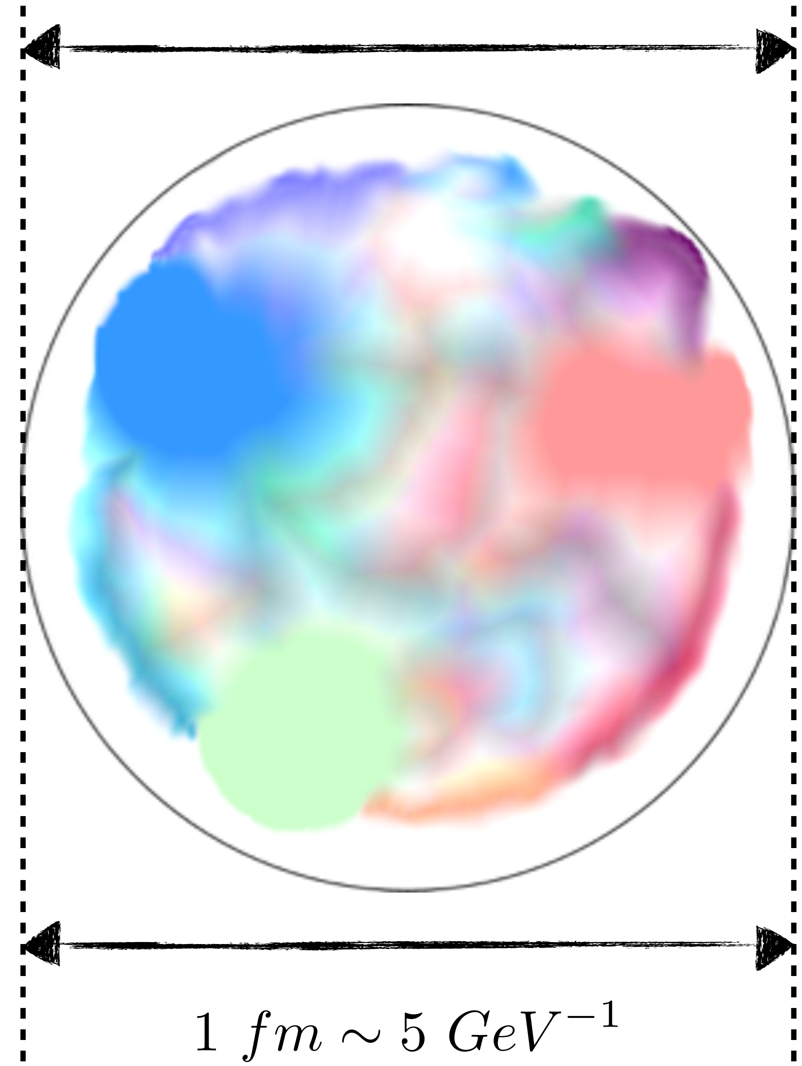



Hadron scale

# Hadron radius (“internal” scale)

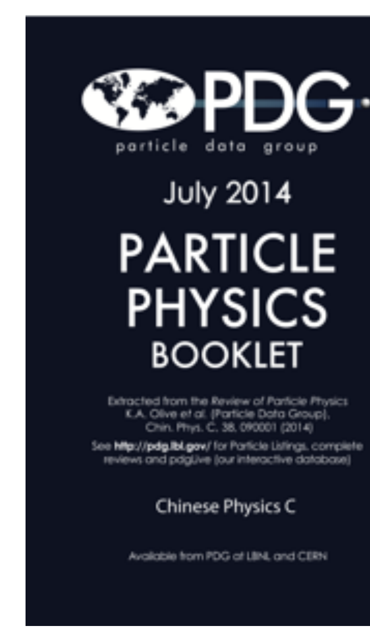



Large coupling constant



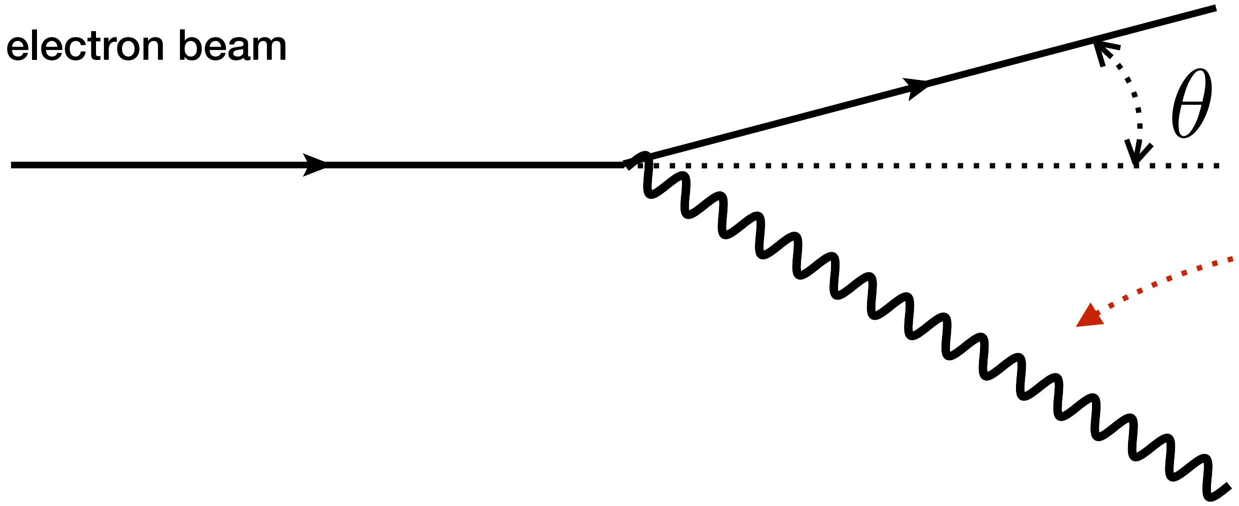

$$1\ fm \sim \frac{1}{0.2\ GeV}$$

Hadron scale

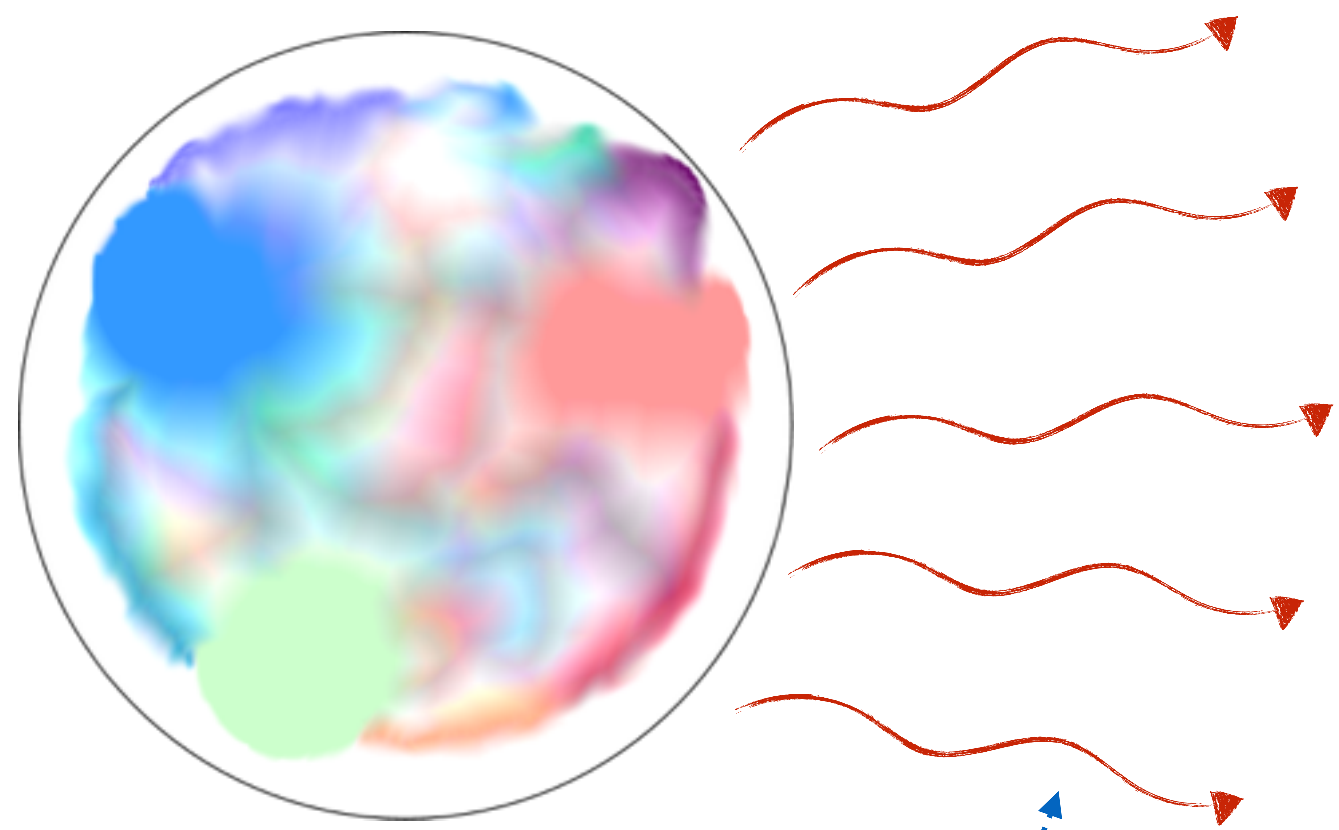



# “External” scale

Deep Inelastic Scattering



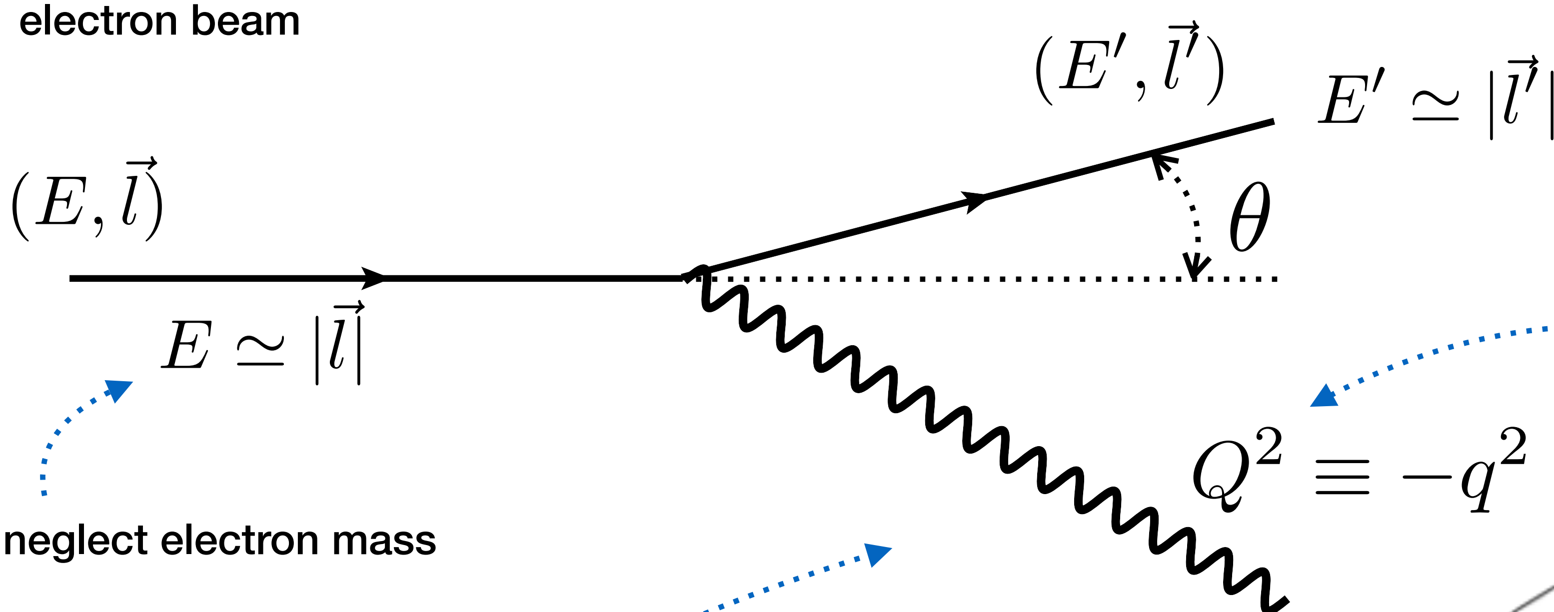

Photon emission (leading QED order)



Inclusive vs. Semi-inclusive processes

# "External" scale

electron beam

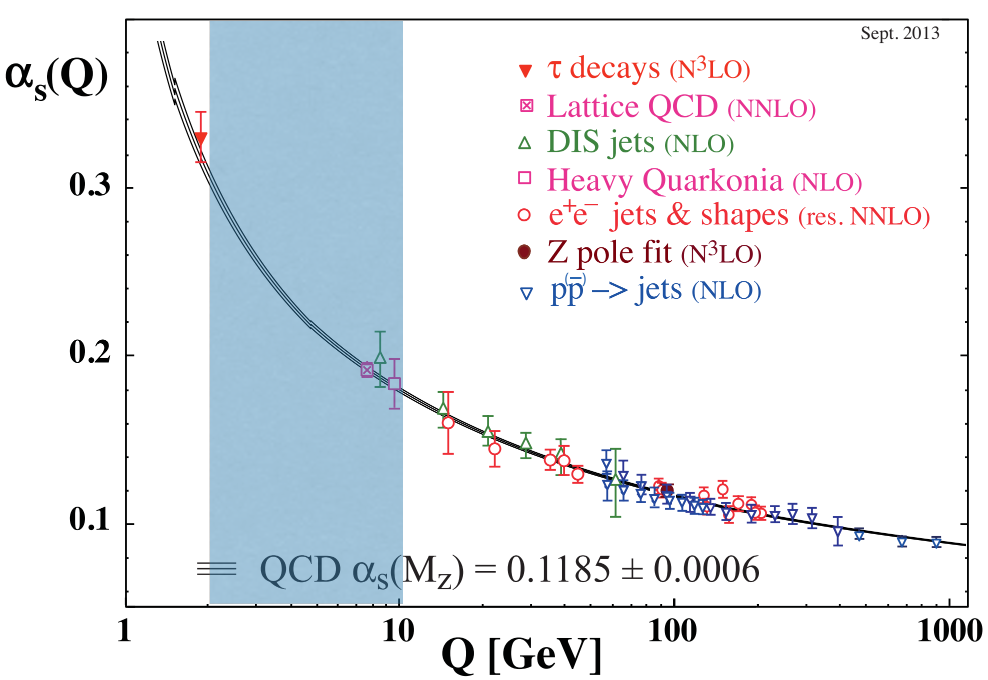


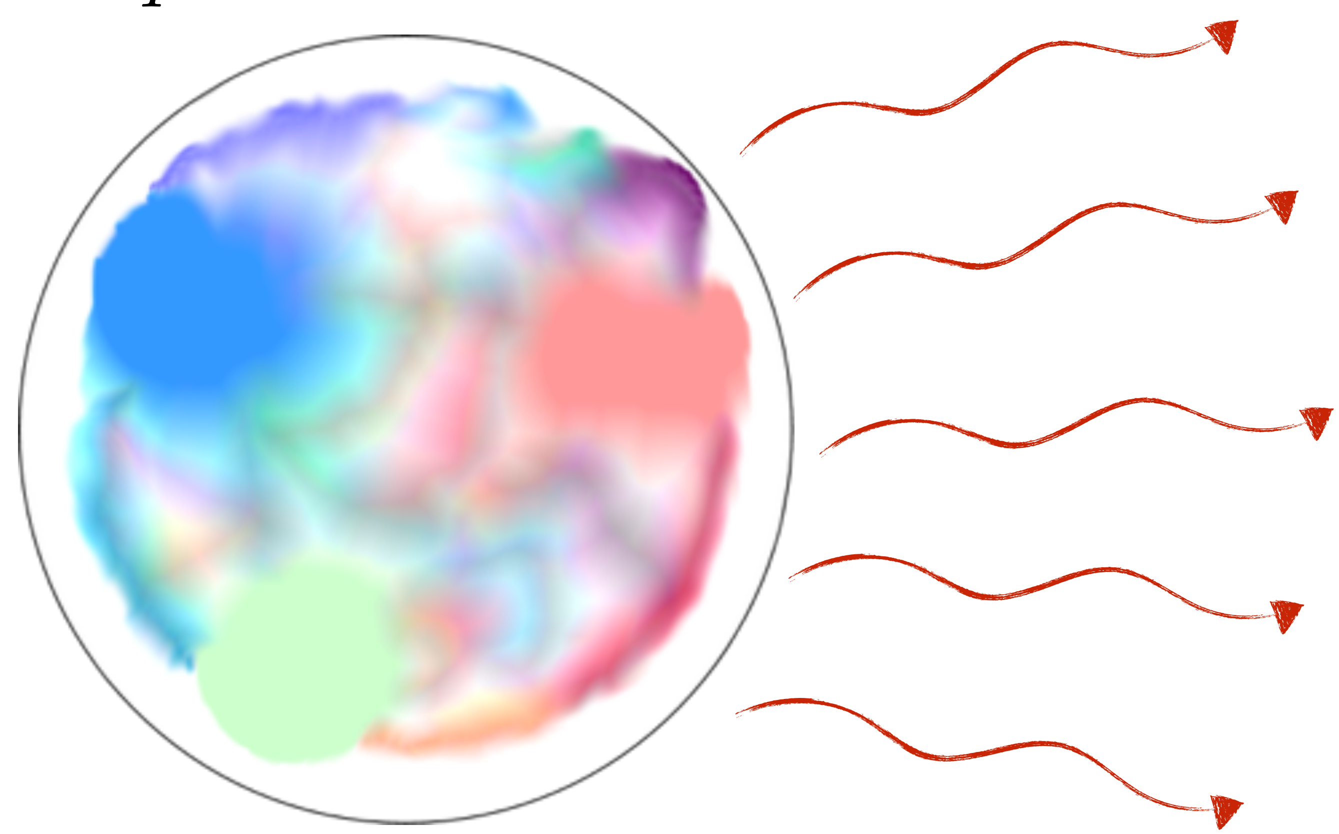

Photon virtuality

neglect electron mass

momentum transfer

$$q^2 \equiv (l' - l)^2 = -2EE'(1 - \cos\theta)$$

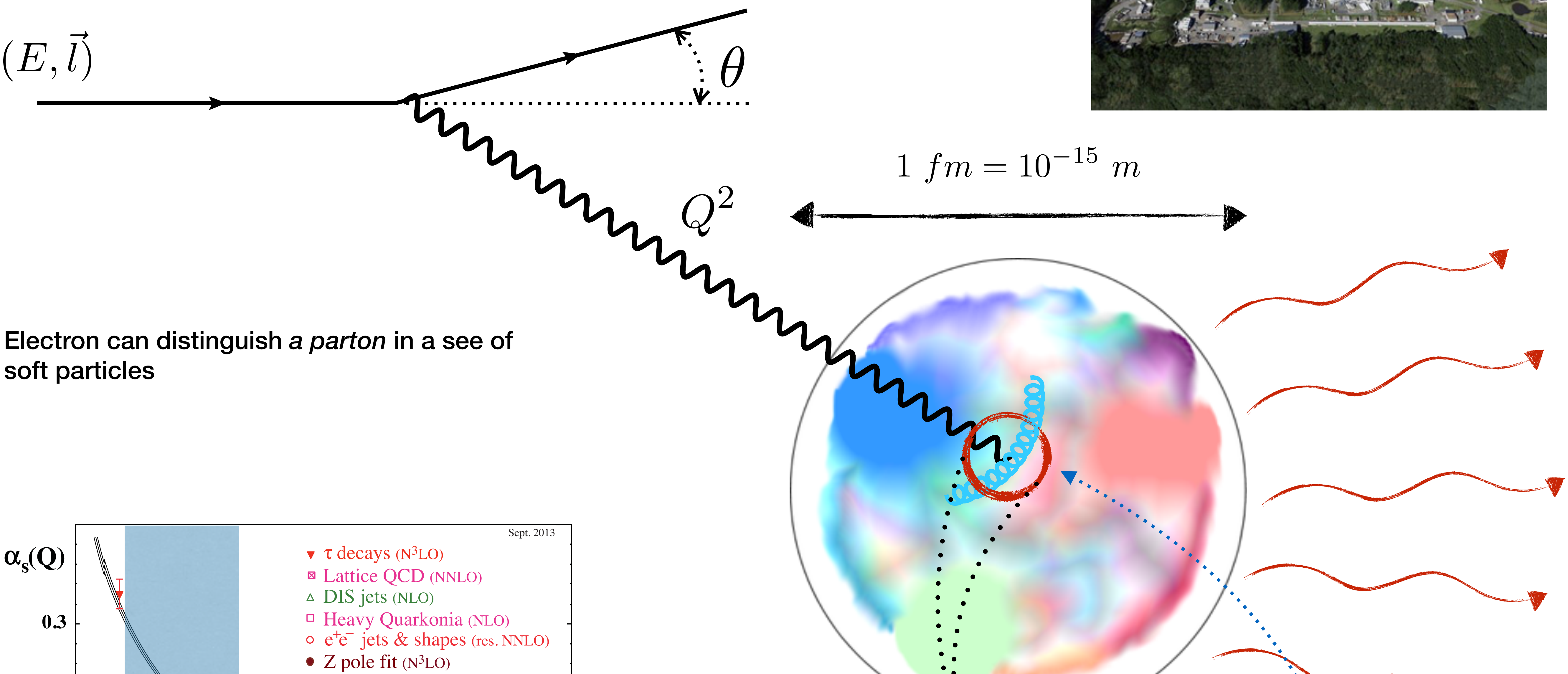


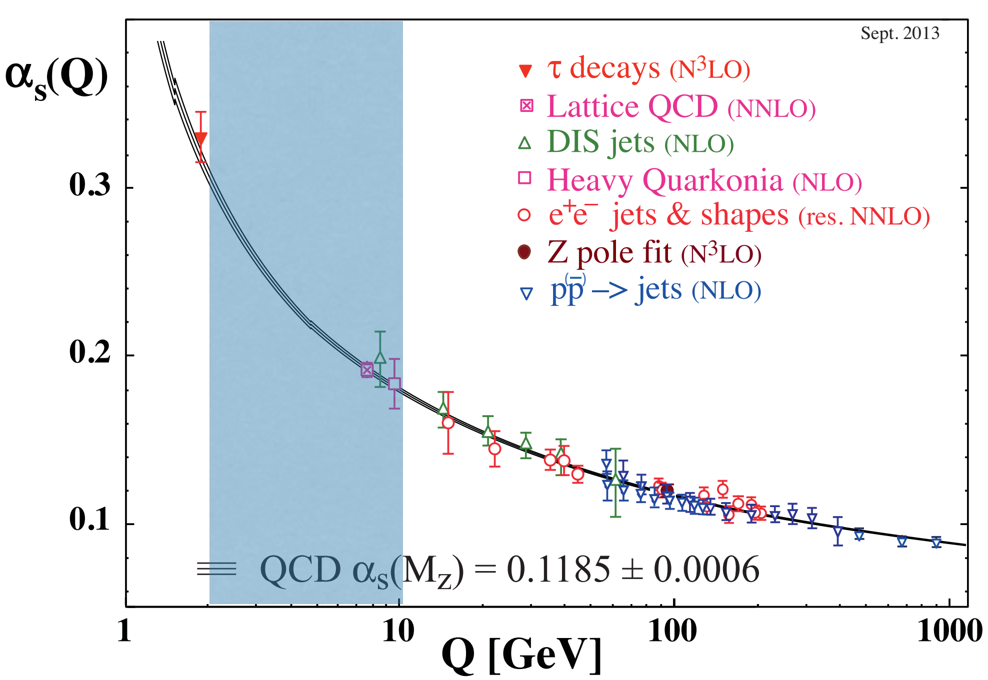
External photon brings a *hard scale* to the problem

# "External" scale

electron beam




Electron can distinguish *a parton* in a see of soft particles

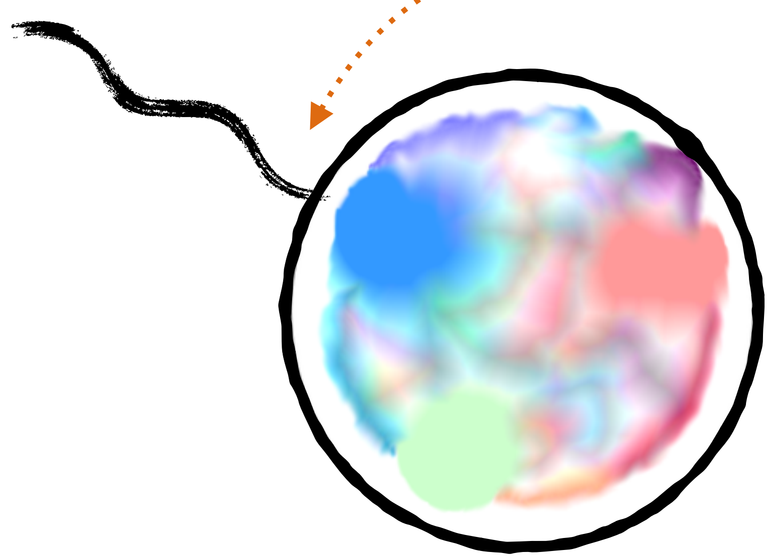



$$\frac{1}{Q} \sim \frac{1}{2\ GeV} \sim 0.1 fm$$

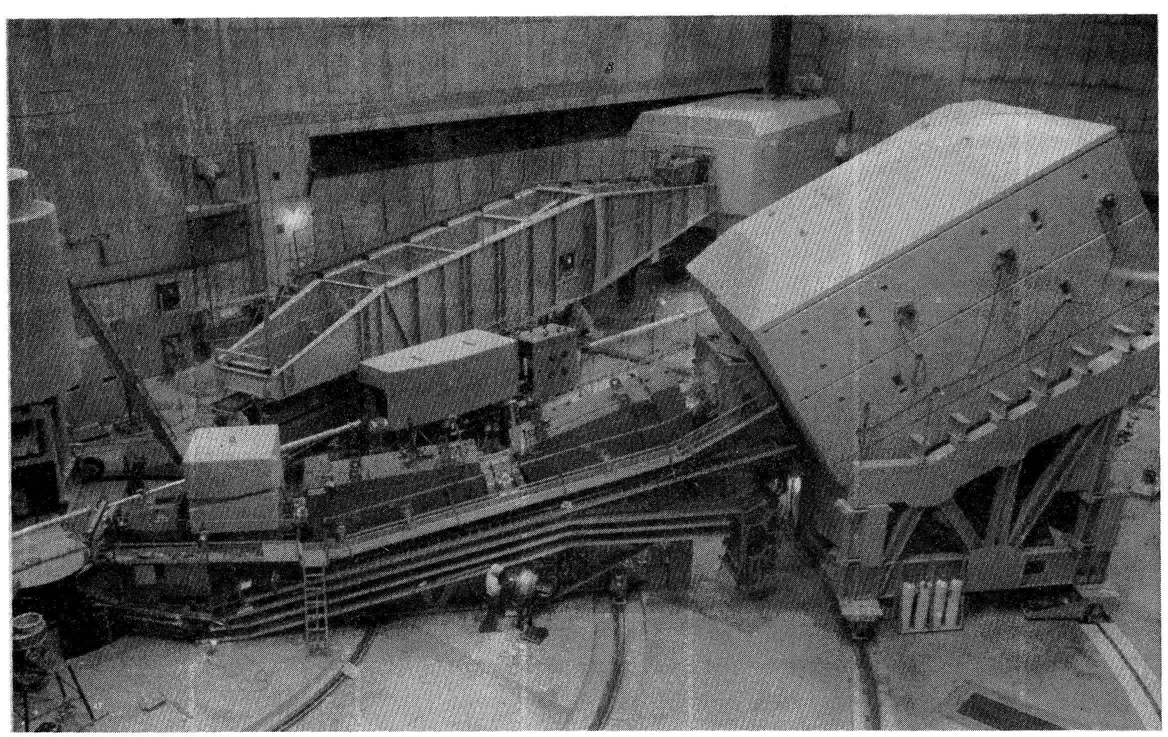
You can see *the parton*

# People didn’t know it 50 years ago

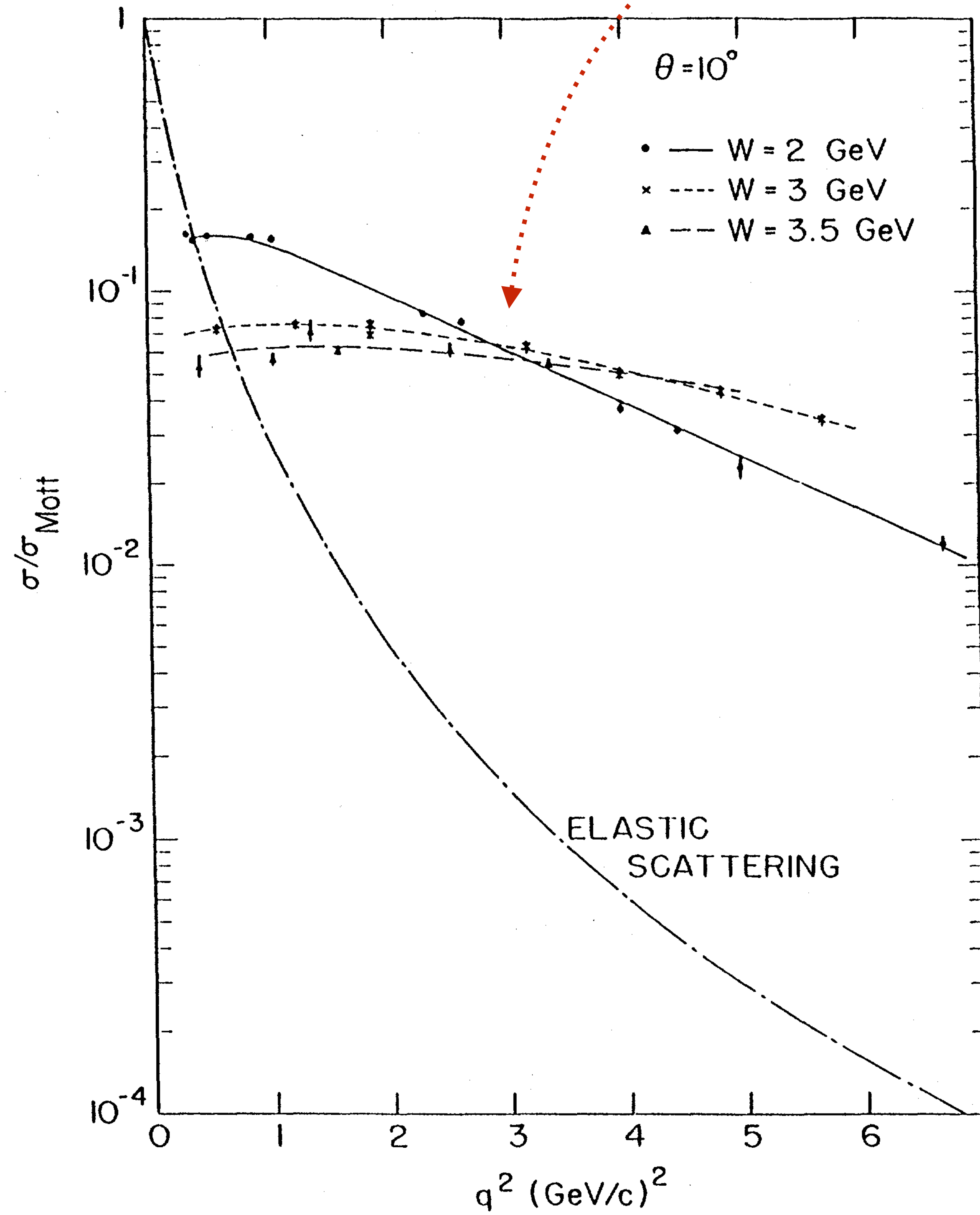
Electron interacts with the a whole hadron



Stanford Linear Accelerator Center (SLAC) in 60s

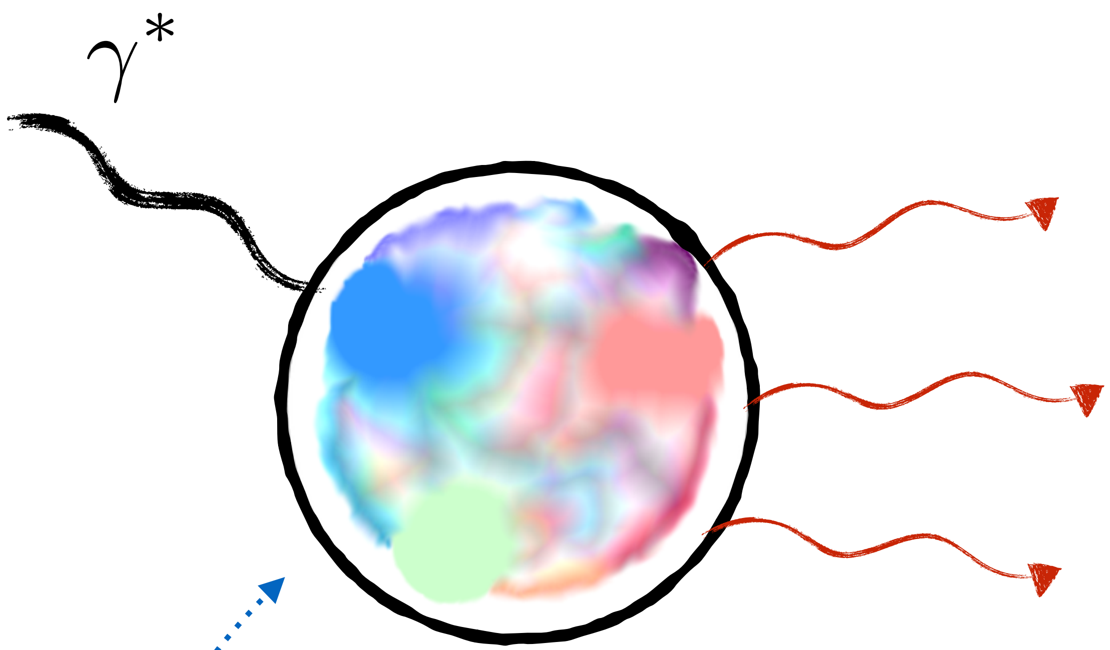


Looks like QED




First evidence of the parton model

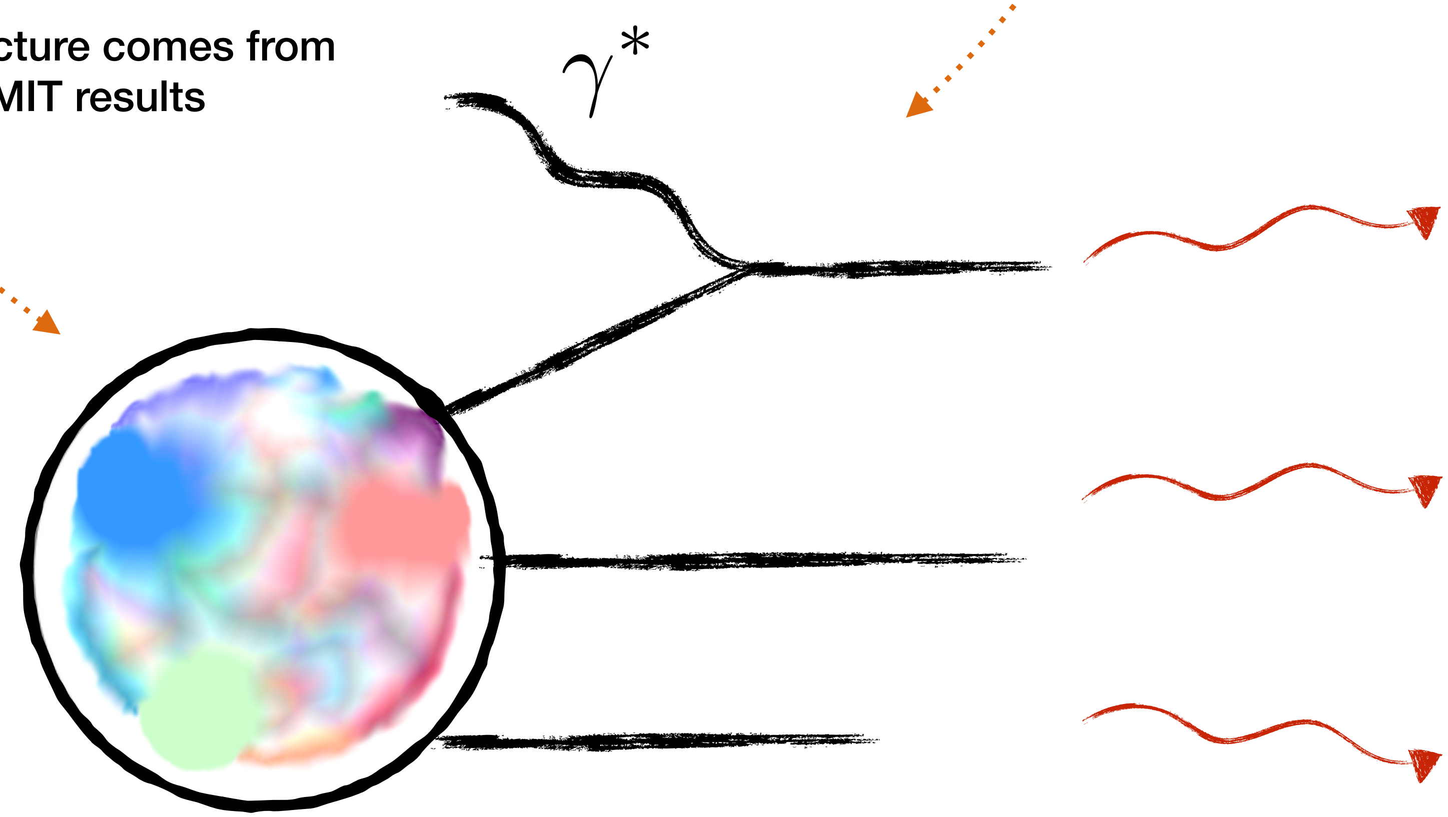
# Parton model



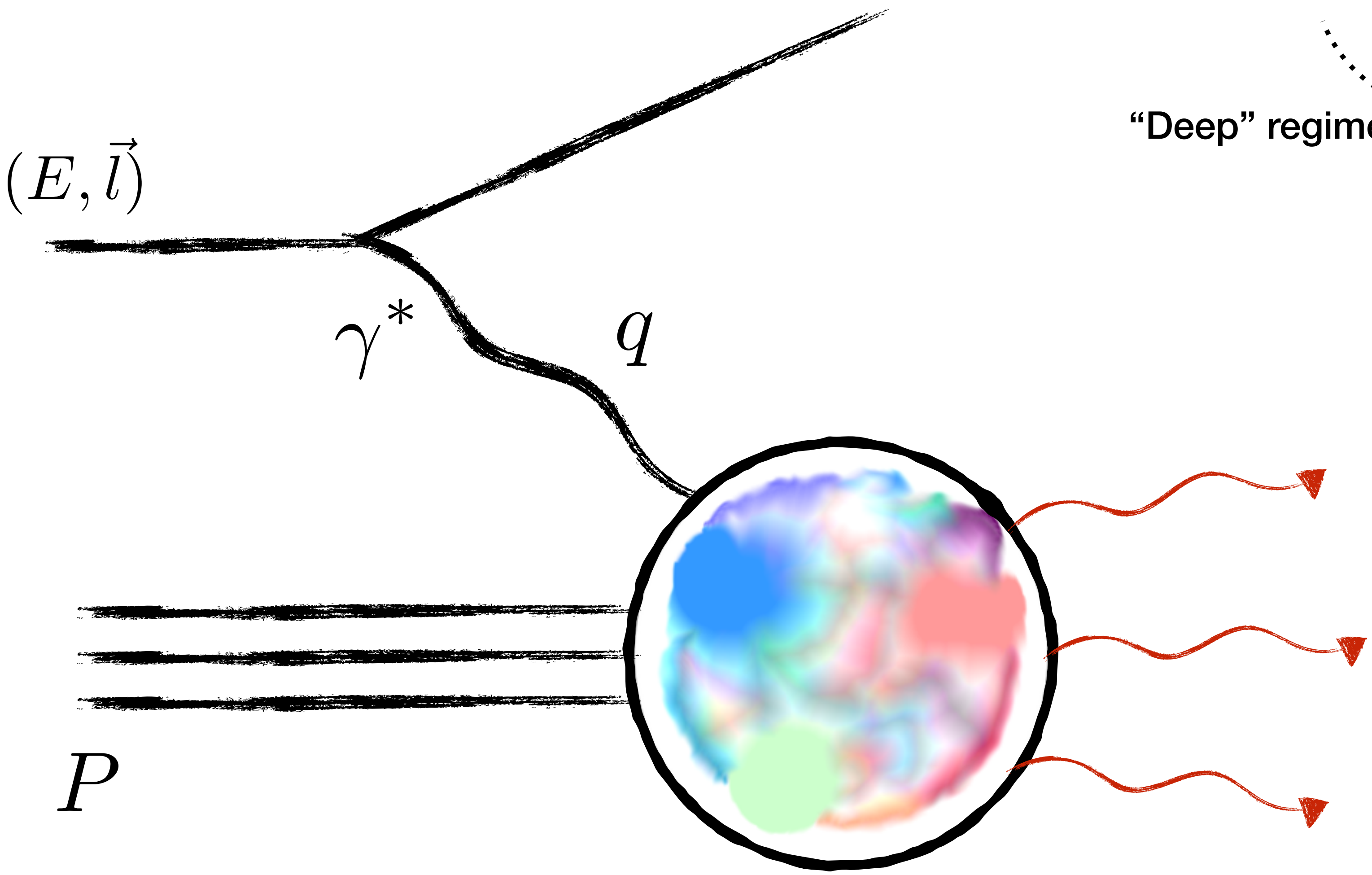

We are in 1965! We don't have the model! Let's start from this.

This picture comes from SLAC-MIT results

In 70s people concluded that QCD is a theory of partons



# DIS kinematics




$Q^2 \gg M^2$

"Deep" regime

Can construct two independent Lorentz invariant variables:

$$Q^2 \equiv -q^2 = 2EE'(1-\cos\theta)$$

$$\nu = \frac{P\cdot q}{M} = E - E'$$

Energy and angle in the lab frame

$$s = (P+l)^2$$

$$W^2 = (P+q)^2$$

$$x = \frac{Q^2}{2P\cdot q} \approx \frac{Q^2}{Q^2+W^2}$$

Bjorken variable

$$y = \frac{P\cdot q}{P\cdot k} \approx \frac{Q^2}{xs}$$

All this variables are scalars

$$f(P_\mu, q_\mu)$$

"Bad" function

vs.

$$f(q^2, P\cdot q)$$

"Good" function

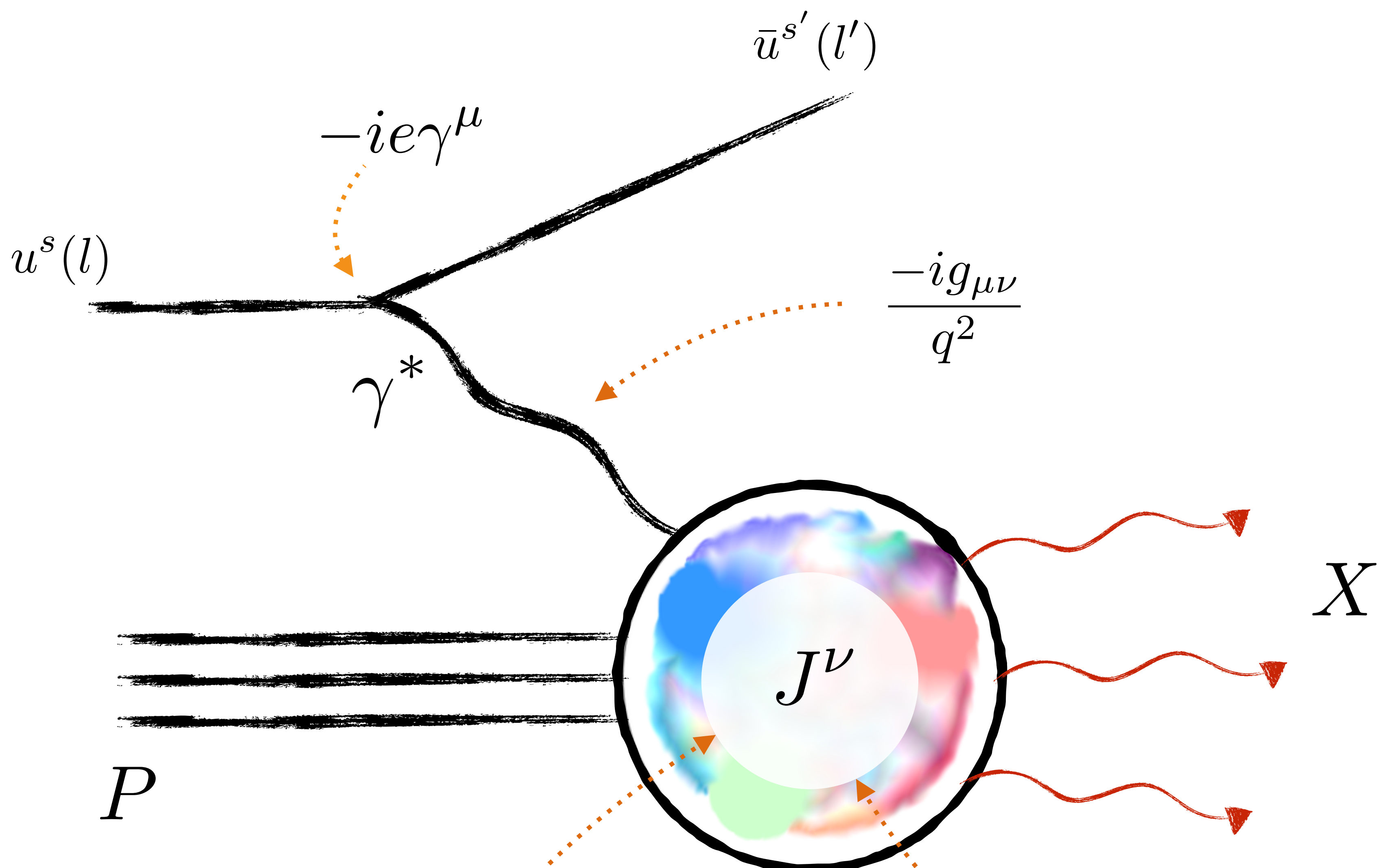
# DIS amplitude

Hadronic current

$$iM = \bar{u}^{s'}(l')(-ie\gamma^{\mu})u^{s}(l) \times \frac{i}{Q^2} \times -ie\langle X|J_{\mu}|P\rangle$$

Leptonic current

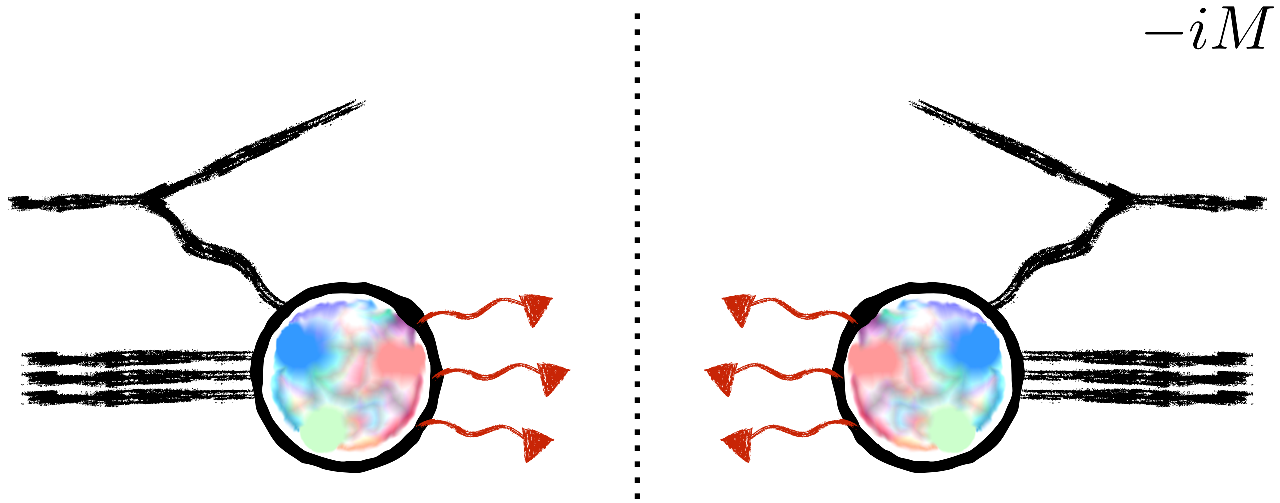
$$q^{\mu}\langle X|J_{\mu}|P\rangle = 0$$




Hadronic current

We don't know QCD so we don't know its structure

# DIS cross section



$$-iM^* = \bar{u}^s(l)(ie\gamma^\nu)u^{s'}(l') \times \frac{-i}{Q^2} \times ie\langle P|J_\nu|X\rangle$$

$$iM = \bar{u}^{s'}(l')(-ie\gamma^\mu)u^s(l) \times \frac{i}{Q^2} \times -ie\langle X|J_\mu|P\rangle$$

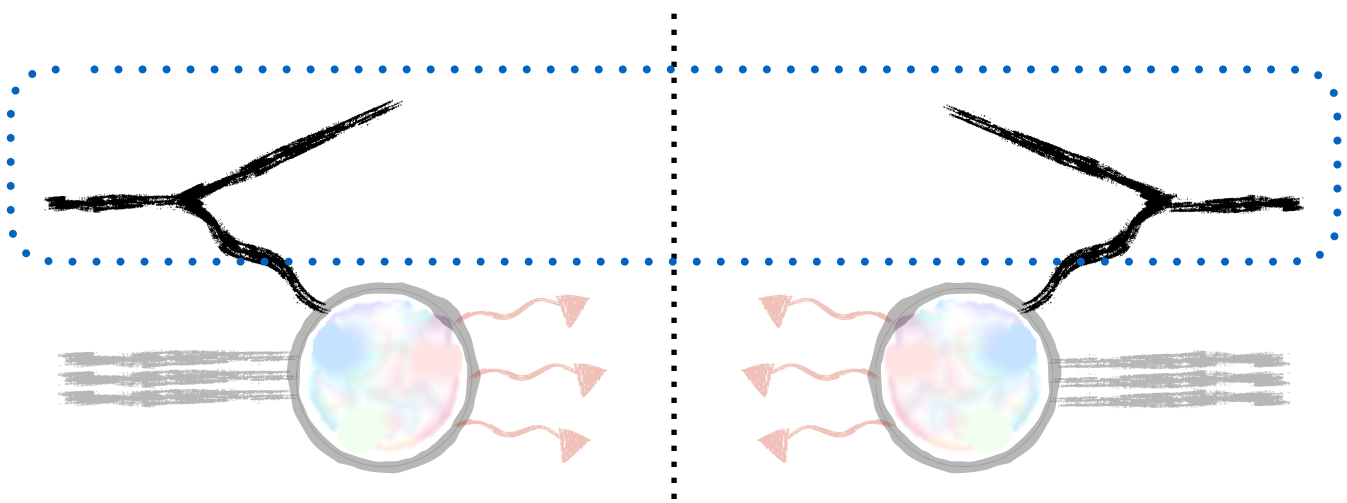
Assume summation over all final states

Compare with SIDIS

$$\frac{1}{2}\frac{1}{2}\sum_{s,s'}|M|^2 = e^4\frac{1}{Q^4} \times \underbrace{\frac{1}{2}\sum_{s,s'}\bar{u}^s_k(l)\gamma^\nu_{kl}u^{s'}_l(l') \times \bar{u}^{s'}_i(l')\gamma^\mu_{ij}u^s_j(l)}_{L^{\mu\nu}} \times \frac{1}{2}\langle P|J_\nu|X\rangle\langle X|J_\mu|P\rangle$$

Average over incoming spin of a hadron and electron

# DIS cross section



Apply completeness relation:

$$L^{\mu\nu} = \frac{1}{2}\sum_{s,s'} \bar{u}_i^{s'}(l')\gamma^{\mu}_{ij}u_j^s(l) \times \bar{u}_k^s(l)\gamma^{\nu}_{kl}u_l^{s'}(l')$$

$$\sum_s u_j^s(l)\bar{u}_k^s(l) = (\not{l} + m)_{jk}$$

sum over spin state

$$L^{\mu\nu} = \frac{1}{2}\not{l}'_{li}\gamma^{\mu}_{ij}\not{l}_{jk}\gamma^{\nu}_{kl} = \frac{1}{2}Tr\{\not{l}'\gamma^{\mu}\not{l}\gamma^{\nu}\}$$

$$Tr\{\text{any odd number of } \gamma\} = 0$$

$$Tr\{\gamma^{\mu}\gamma^{\nu}\} = 4g^{\mu\nu}$$

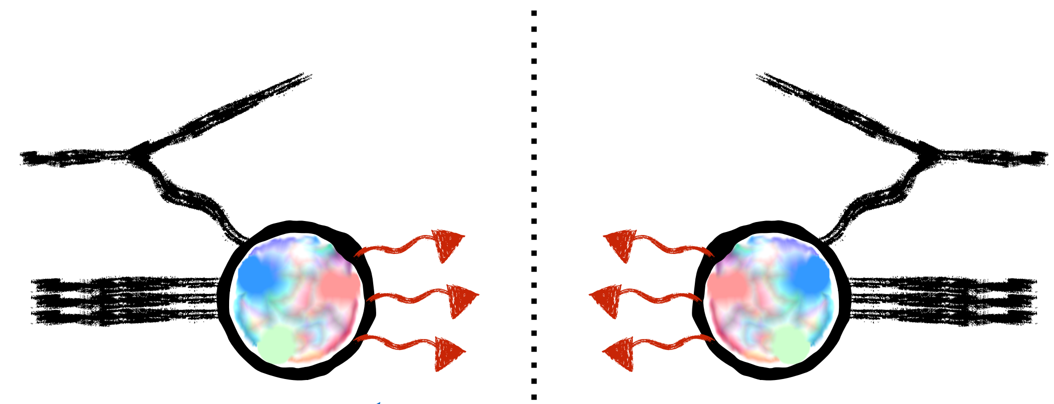
$$Tr\{\gamma^{\mu}\gamma^{\nu}\gamma^{\rho}\gamma^{\sigma}\} = 4\big(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho}\big)$$

$$L^{\mu\nu} = 2(l'^{\mu}l^{\nu} + l'^{\nu}l^{\mu} - g^{\mu\nu}l\cdot l')$$

Final answer for the leptonic part. Pretty easy to get!!!

# It is time to write the cross section

$$d\sigma = \frac{1}{4}[(k_A \cdot k_B)^2 - m_A^2 m_B^2]^{-1/2}$$
$$\times \prod_i \int \frac{d^3 p_i}{(2\pi)^3 2E_i} |M|^2 (2\pi)^4 \delta^4(k_A + k_B - \sum p_i)$$



We don't detect this particles in the final state

Formula for the cross section from the previous lecture

final state phase space

$$\frac{d^3 l'}{(2\pi)^3 2E'} dX$$

$$d\sigma = \frac{1}{2(S - M^2)} \frac{1}{4} \sum_{s,s'} |M|^2 \frac{d^3 l'}{(2\pi)^3 2E'} dX (2\pi)^4 \delta^4(P + q - X)$$

flux factor

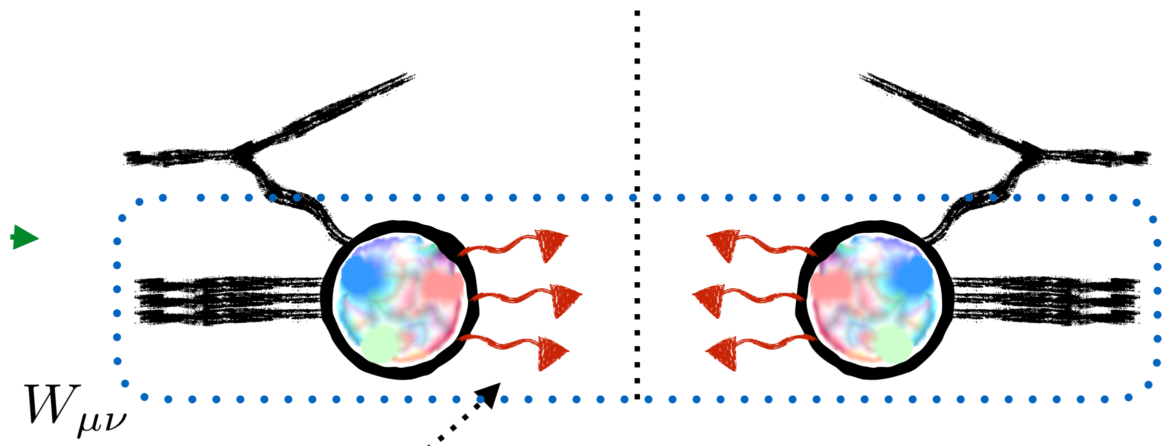
$$\frac{1}{4} \sum_{s,s'} |M|^2 = e^4 \frac{1}{Q^4} L^{\mu\nu} \frac{1}{2} \langle P|J_\nu|X\rangle \langle X|J_\mu|P\rangle$$

Calculated this on the previous slide

Do not know what it is

# It is time to write the cross section

"Inclusive" process




All possible final states

Unknown hadronic part

$$d\sigma = \frac{1}{2(S-M^2)} e^4 \frac{1}{Q^4} L^{\mu\nu} \times \frac{1}{2} \int dX \langle P|J_\nu|X\rangle\langle X|J_\mu|P\rangle (2\pi)^4 \delta^4(P+q-X) \times \frac{d^3l'}{(2\pi)^3 2E'}$$

Hadronic tensor

$$W_{\mu\nu} = \frac{1}{2}\frac{1}{4\pi M} \int dX \langle P|J_\nu|X\rangle\langle X|J_\mu|P\rangle (2\pi)^4 \delta^4(P+q-X)$$

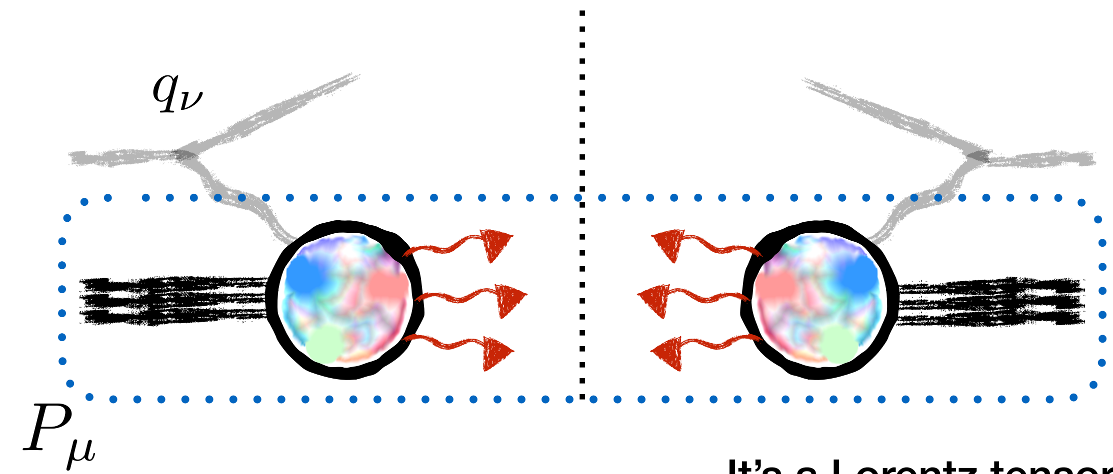
Final form of the cross section

$$d\sigma = \frac{1}{2(S-M^2)} e^4 \frac{1}{Q^4} L^{\mu\nu} 4\pi M W_{\mu\nu} \frac{d^3l'}{(2\pi)^3 2E'}$$

We have no information about this object

# Hadronic tensor

$$W_{\mu\nu} = \frac{1}{2}\frac{1}{4\pi M}\int dX \langle P|J_\nu|X\rangle\langle X|J_\mu|P\rangle(2\pi)^4\delta^4(P+q-X)$$




It's a Lorentz tensor

Can "construct" tensors:

$P_\mu P_\nu$ $\quad P_\mu q_\nu$ $\quad P_\nu q_\mu$ $\quad q_\mu q_\nu$ $\quad g_{\mu\nu}$

Combine

?

$$\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) \quad \left(P_\mu - q_\mu\frac{P\cdot q}{q^2}\right) \quad \left(P_\nu - q_\nu\frac{P\cdot q}{q^2}\right)$$

Limited number of structures

Hadronic current conservation

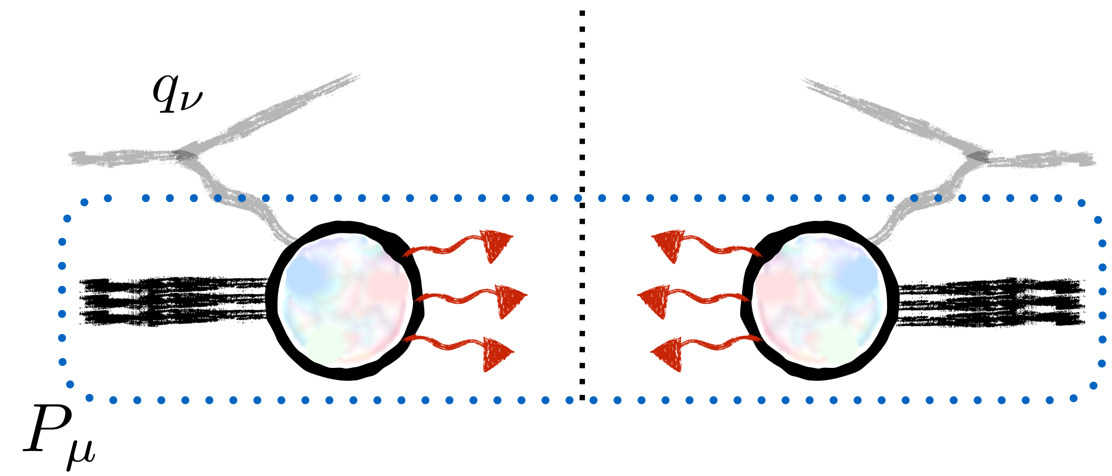
$$q^\mu W_{\mu\nu} = 0 \qquad q^\nu W_{\mu\nu} = 0$$

The only thing we know about hadronic tensor

Let's find functions, which fulfill this condition

# Hadronic tensor

$$W_{\mu\nu} = \frac{1}{2}\frac{1}{4\pi M}\int dX \langle P|J_\nu|X\rangle\langle X|J_\mu|P\rangle(2\pi)^4\delta^4(P+q-X)$$




Structure functions

Scalar coefficients

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)W_1(\nu, Q^2) + \frac{1}{M^2}\left(P_\mu - q_\mu\frac{P\cdot q}{q^2}\right)\left(P_\nu - q_\nu\frac{P\cdot q}{q^2}\right)W_2(\nu, Q^2)$$

$$d\sigma = \frac{1}{2(S-M^2)}e^4\frac{1}{Q^4}L^{\mu\nu}4\pi M W_{\mu\nu}\frac{d^3l'}{(2\pi)^3 2E'}$$

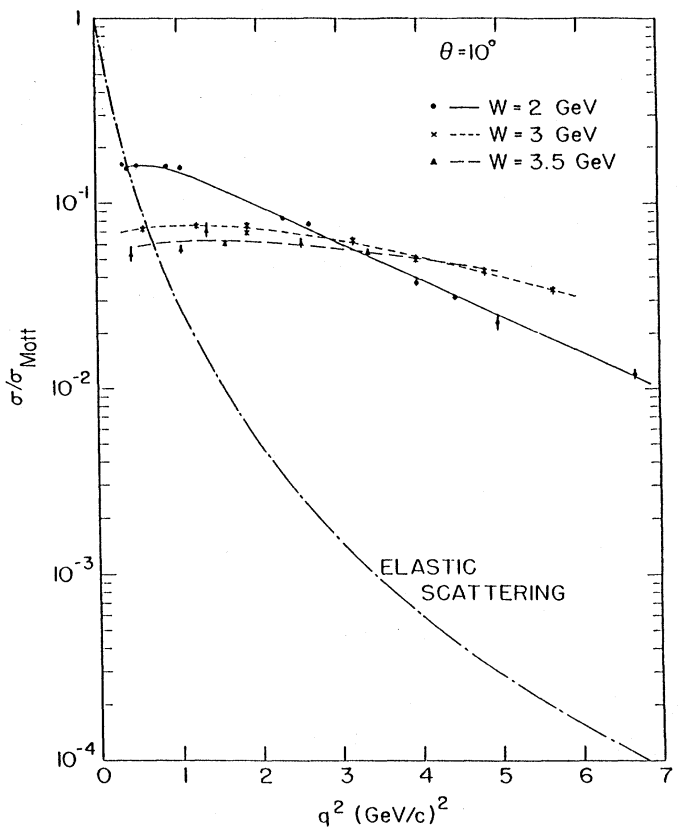
$$F_1(x, Q^2) = 2MW_1(\nu, Q^2)$$

$$F_2(x, Q^2) = \nu W_2(\nu, Q^2)$$

Jefferson Lab
Thomas Jefferson National Accelerator Facility

Measure this functions
(that is the whole story:)

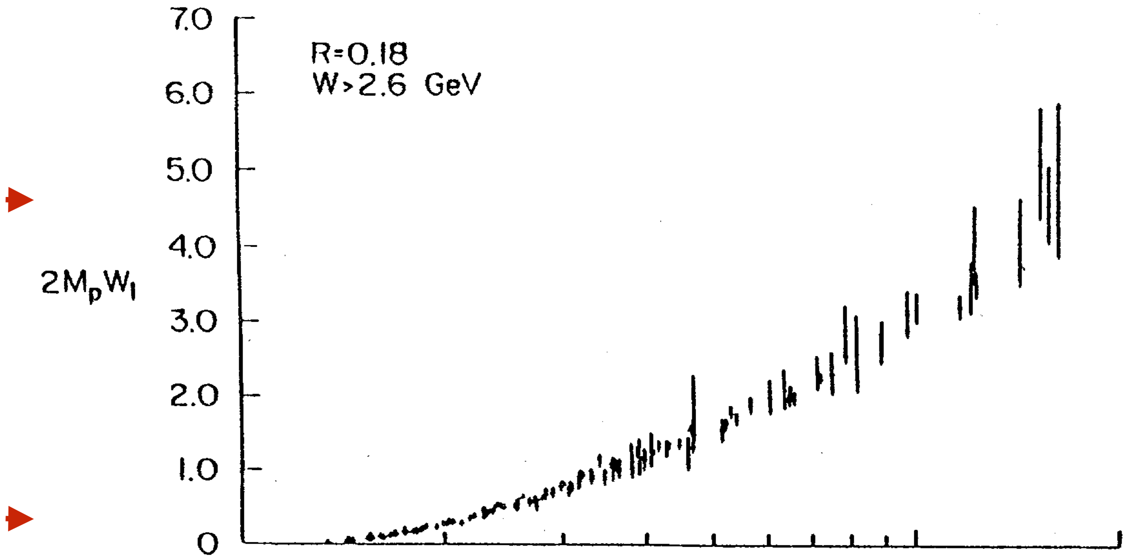
# SLAC-MIT Collaboration

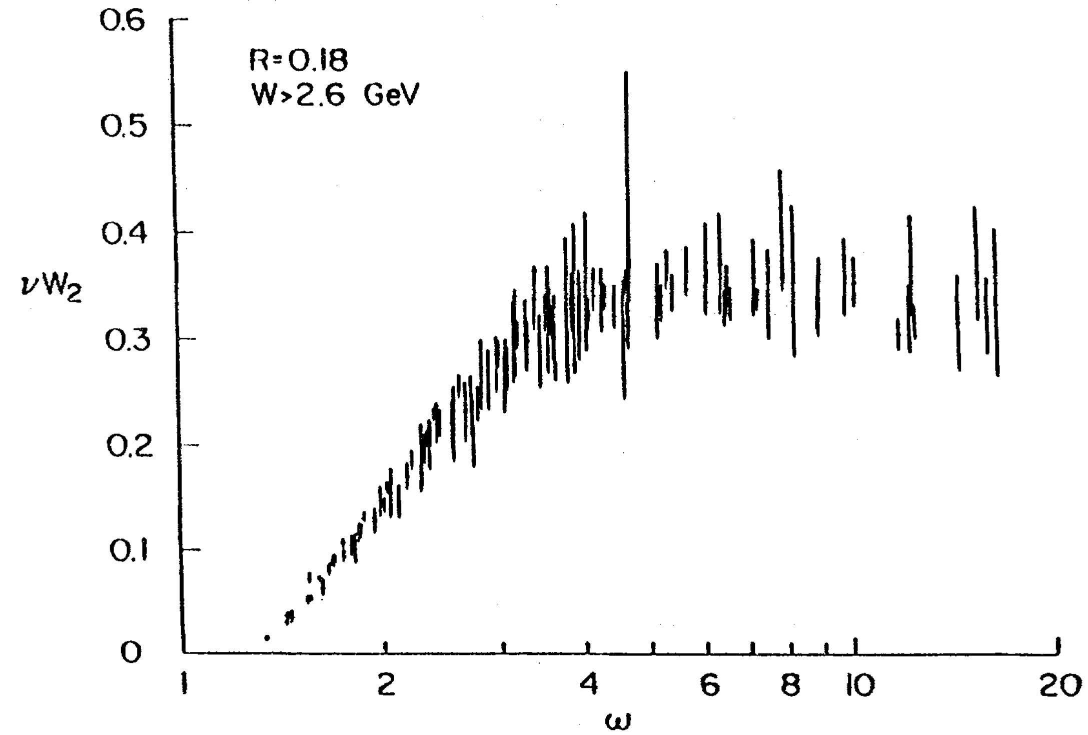


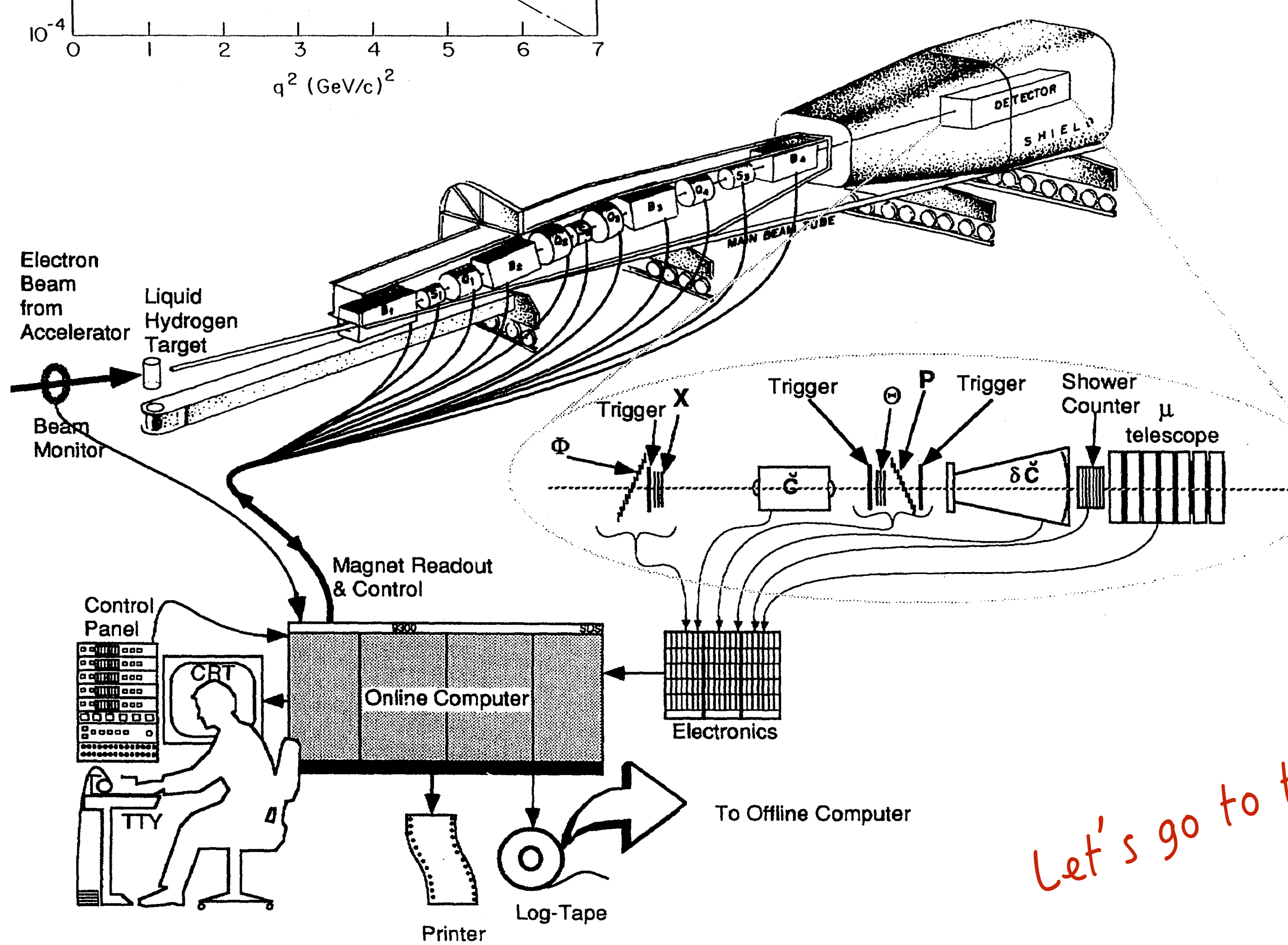

Predicted by Bjorken

Explained by Feynman within the parton model

$2\ GeV^2 < Q^2 < 20\ GeV^2$



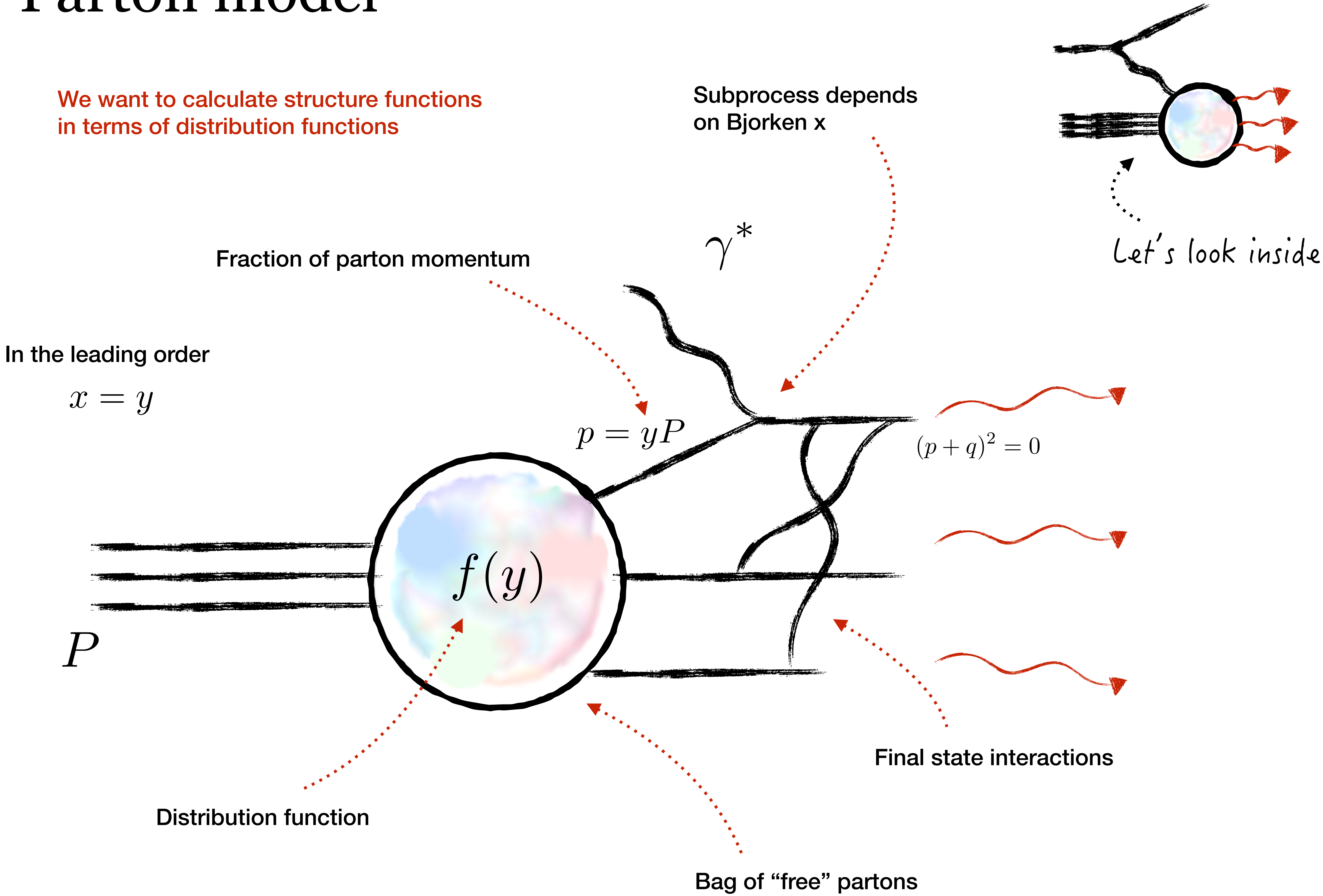

*Let's go to the parton model*

$$F_1(x, Q^2) = 2MW_1(\nu, Q^2)$$

$$F_2(x, Q^2) = \nu W_2(\nu, Q^2)$$

# Parton model

We want to calculate structure functions
in terms of distribution functions

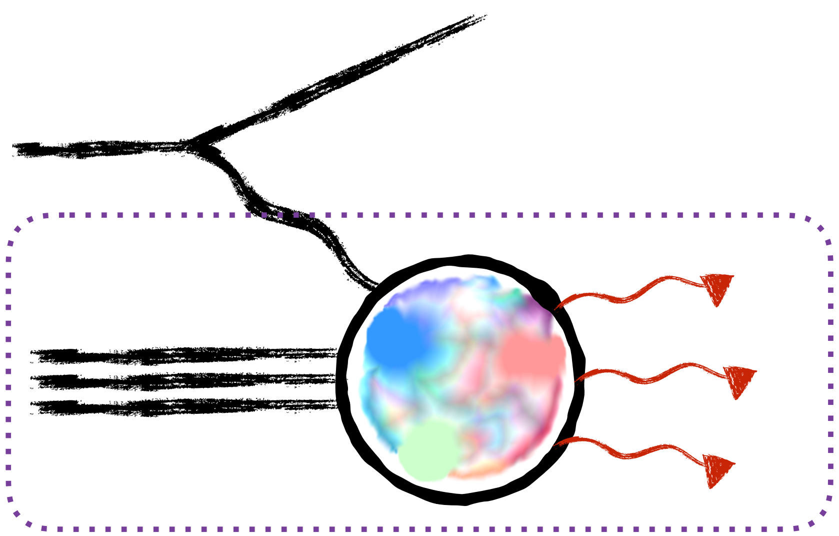


# Factorization

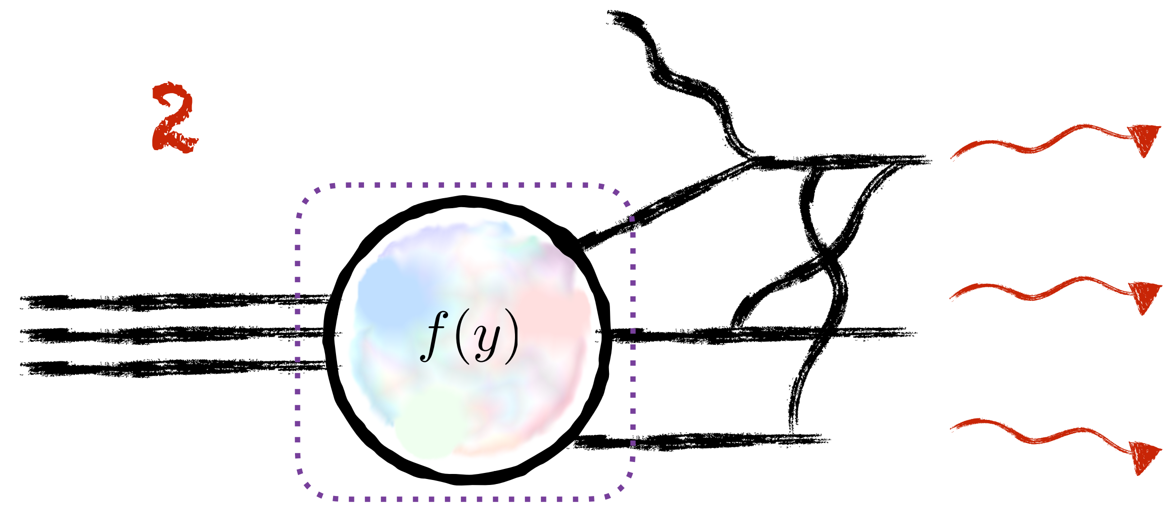
Information on the hadron

**1**

$$d\sigma = \frac{1}{2S} e^4 \frac{1}{Q^4} L^{\mu\nu} 4\pi M W_{\mu\nu} \frac{d^3 l'}{(2\pi)^3 2E'}$$



**2**




Information on the hadron

$$d\sigma = \int_x^1 dy f(y) d\tilde{\sigma}$$

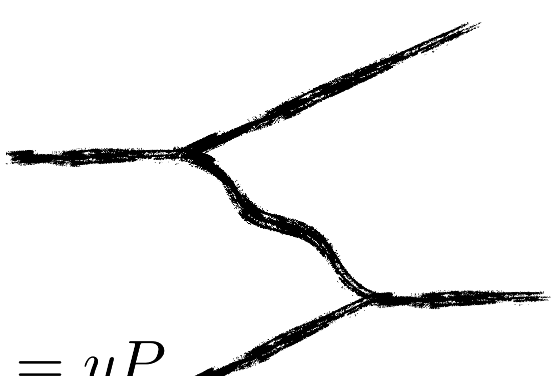
$(p+q)^2 \geq 0$

Scattering of the photon on a single quark

**3**

$$d\tilde{\sigma} = \frac{1}{2s} e^4 \frac{1}{Q^4} L^{\mu\nu} \tilde{W}_{\mu\nu} \frac{d^3 l'}{(2\pi)^3 2E'}$$

$s = yS$



$p = yP$

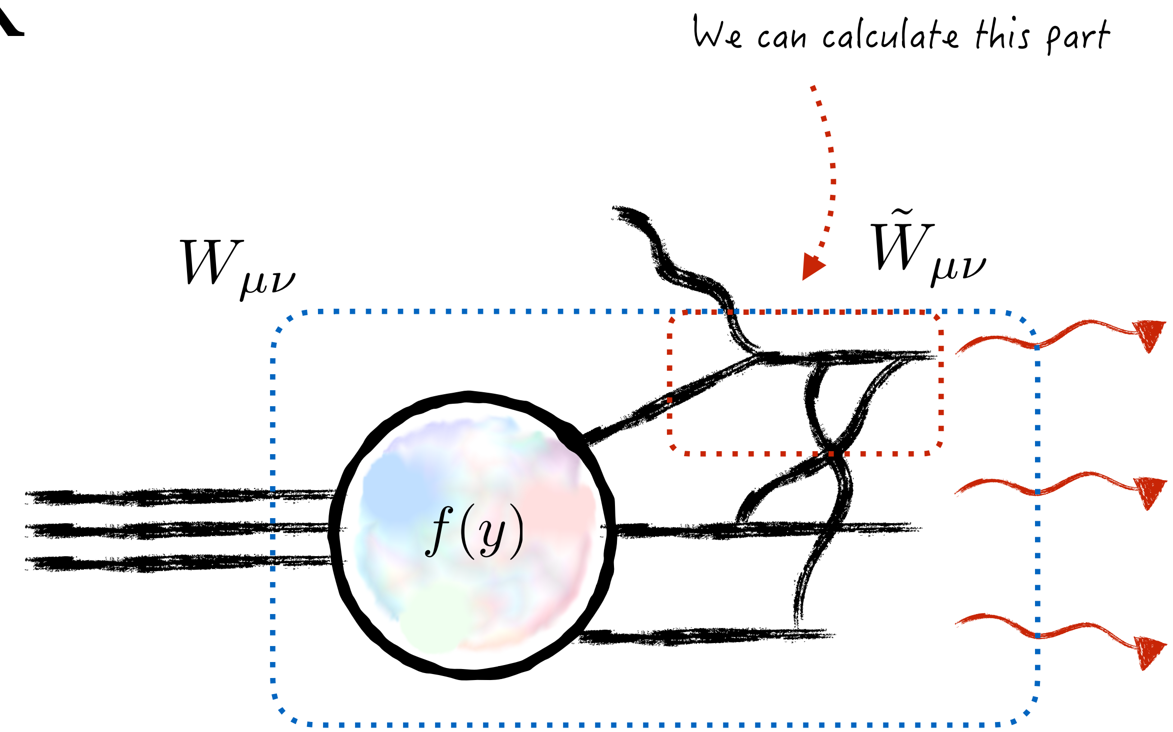
$$\tilde{W}_{\mu\nu} = \frac{1}{2} \int dX \langle p | J_\nu | X \rangle \langle X | J_\mu | p \rangle (2\pi)^4 \delta^4(p + q - X)$$

# Scattering on a single quark

$$W_{\mu\nu} = \frac{1}{4\pi M}\int_x^1 \frac{dy}{y} f(y)\tilde{W}_{\mu\nu}$$

Non-perturbative part

Perturbative part




$$\tilde{W}_{\mu\nu} = \frac{1}{2}\int dX \langle p|J_\nu|X\rangle\langle X|J_\mu|p\rangle(2\pi)^4\delta^4(p+q-X)$$

$$\frac{d^3p'}{(2\pi)^3 2E'}$$

$\langle p'|$

$J_\nu = Q_i\bar{\psi}\gamma_\mu\psi$ Quark current

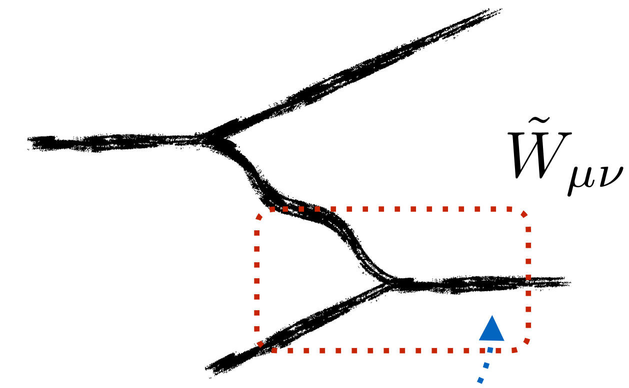
$$\tilde{W}_{\mu\nu} = \frac{1}{2}Q_i^2\int \frac{d^3p'}{(2\pi)^3 2E'} \times \bar{u}_i^s(p)\gamma_\nu^{ij}u_j^{s'}(p') \times \bar{u}_k^{s'}(p')\gamma_\mu^{kl}u_l^s(p) \times (2\pi)^4\delta^4(p+q-p')$$

$$\sum_s u_l^s(p)\bar{u}_i^s(p) = (\not{p}+m)_{li}$$

$$\tilde{W}_{\mu\nu} = \frac{1}{2}Q_i^2\int \frac{d^3p'}{(2\pi)^3 2E'} \times Tr\{\not{p}'\gamma_\mu\not{p}\gamma_\nu\} \times (2\pi)^4\delta^4(p+q-p')$$

# Scattering on a single quark

$$\tilde{W}_{\mu\nu} = \frac{1}{2} Q_i^2 \int \frac{d^3p'}{(2\pi)^3 2E'} \times Tr\{\not{p}'\gamma_\mu \not{p}\gamma_\nu\} \times (2\pi)^4 \delta^4(p+q-p')$$




Let's modify the phase-space integration:

The outgoing quark is on the mass-shelf

$$\int \frac{d^3p'}{(2\pi)^3 2E'} \times (2\pi)^4 \delta^4(p+q-p') = \int \frac{d^4p'}{(2\pi)^4} \times 2\pi\delta(p'^2) \times (2\pi)^4 \delta^4(p+q-p')$$

Integrate over momentum conservation

$$\int \frac{d^3p'}{(2\pi)^3 2E'} \times (2\pi)^4 \delta^4(p+q-p') = 2\pi\delta\Big(2p\cdot q - Q^2\Big)$$

In the leading order two variables coincide

$$\tilde{W}_{\mu\nu} = 2Q_i^2 \Big(2y^2 P_\mu P_\nu + y q_\mu P_\nu + y q_\nu P_\mu - y g_{\mu\nu} P\cdot q\Big) \frac{2\pi\delta(y-x)}{2P\cdot q}$$

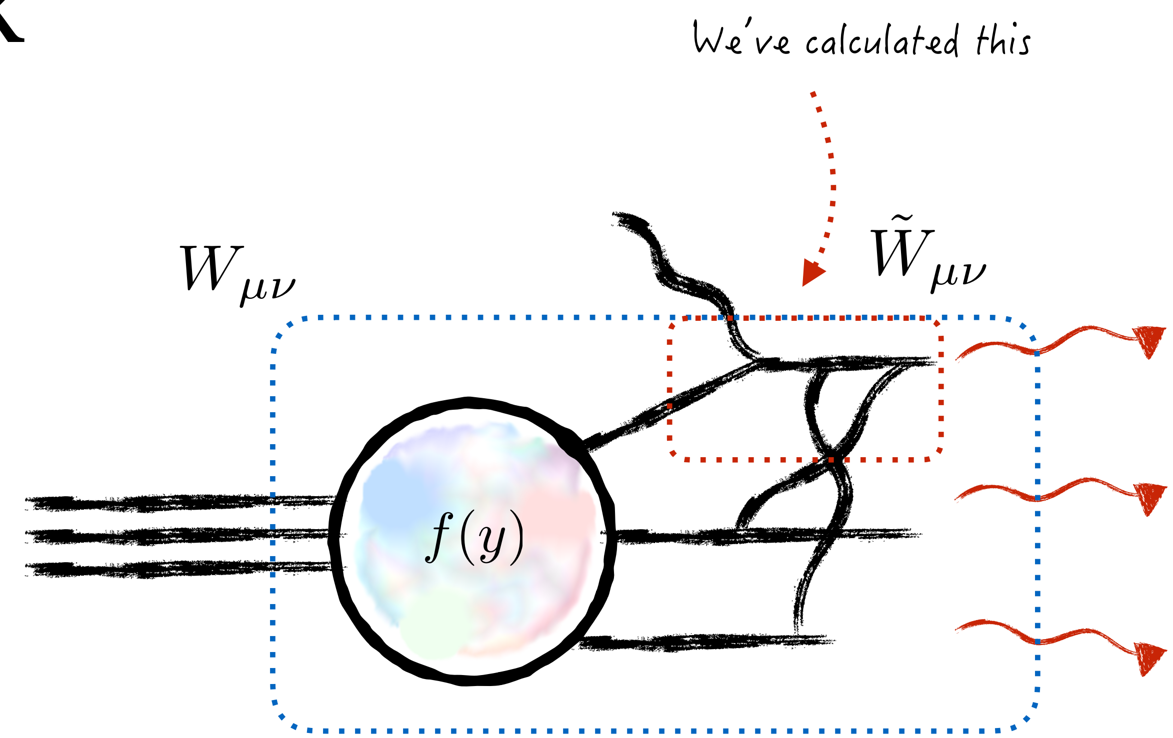
We've got an explicit formula for the photon scattering on a single quark

# Scattering on a single quark

$$W_{\mu\nu} = \frac{1}{4\pi M} \int_x^1 \frac{dy}{y} f(y) \tilde{W}_{\mu\nu}$$

Non-perturbative part

Perturbative part




$$\tilde{W}_{\mu\nu} = 2Q_i^2 \left(2y^2 P_\mu P_\nu + y q_\mu P_\nu + y q_\nu P_\mu - y g_{\mu\nu} P \cdot q\right) \frac{2\pi\delta(y-x)}{2P \cdot q}$$

Sum over different flavors

$$W_{\mu\nu} = \frac{1}{4\pi M} \frac{2\pi}{P \cdot q} \sum_i Q_i^2 \int_x^1 \frac{dy}{y} f_i(y) \left(2y^2 P_\mu P_\nu + y q_\mu P_\nu + y q_\nu P_\mu - y g_{\mu\nu} P \cdot q\right) \delta(y-x)$$

$$x = \frac{Q^2}{2\nu M}$$

Compare:

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W_1(\nu, Q^2) + \frac{1}{M^2} \left(P_\mu - q_\mu \frac{P \cdot q}{q^2}\right)\left(P_\nu - q_\nu \frac{P \cdot q}{q^2}\right) W_2(\nu, Q^2)$$

# Scattering on a single quark

$$W_{\mu\nu} = \frac{1}{4\pi M}\frac{2\pi}{P\cdot q}\sum_i Q_i^2 \int_x^1 \frac{dy}{y} f_i(y)\Big(2y^2 P_\mu P_\nu + y q_\mu P_\nu + y q_\nu P_\mu - y g_{\mu\nu} P\cdot q\Big)\delta(y-x)$$

Rewrite

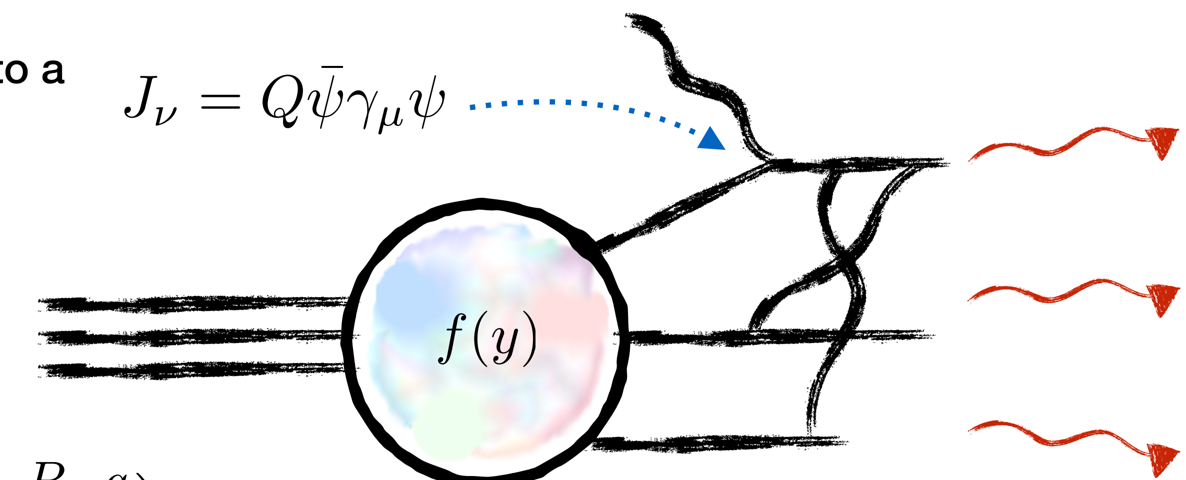
$$x = \frac{Q^2}{2\nu M} \qquad \nu = \frac{P\cdot q}{M}$$

$$W_{\mu\nu} = \sum_i Q_i^2 f_i(x)\Big\{-\Big(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\Big)\frac{1}{2M} + \frac{1}{M^2}\Big(P_\mu - q_\mu\frac{P\cdot q}{q^2}\Big)\Big(P_\nu - q_\nu\frac{P\cdot q}{q^2}\Big)\frac{Q^2}{2\nu P\cdot q}\Big\}$$

Compare

Quark current leads to a proper form

$$J_\nu = Q\bar{\psi}\gamma_\mu\psi$$




$$W_{\mu\nu} = -\Big(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\Big)W_1(\nu,Q^2) + \frac{1}{M^2}\Big(P_\mu - q_\mu\frac{P\cdot q}{q^2}\Big)\Big(P_\nu - q_\nu\frac{P\cdot q}{q^2}\Big)W_2(\nu,Q^2)$$

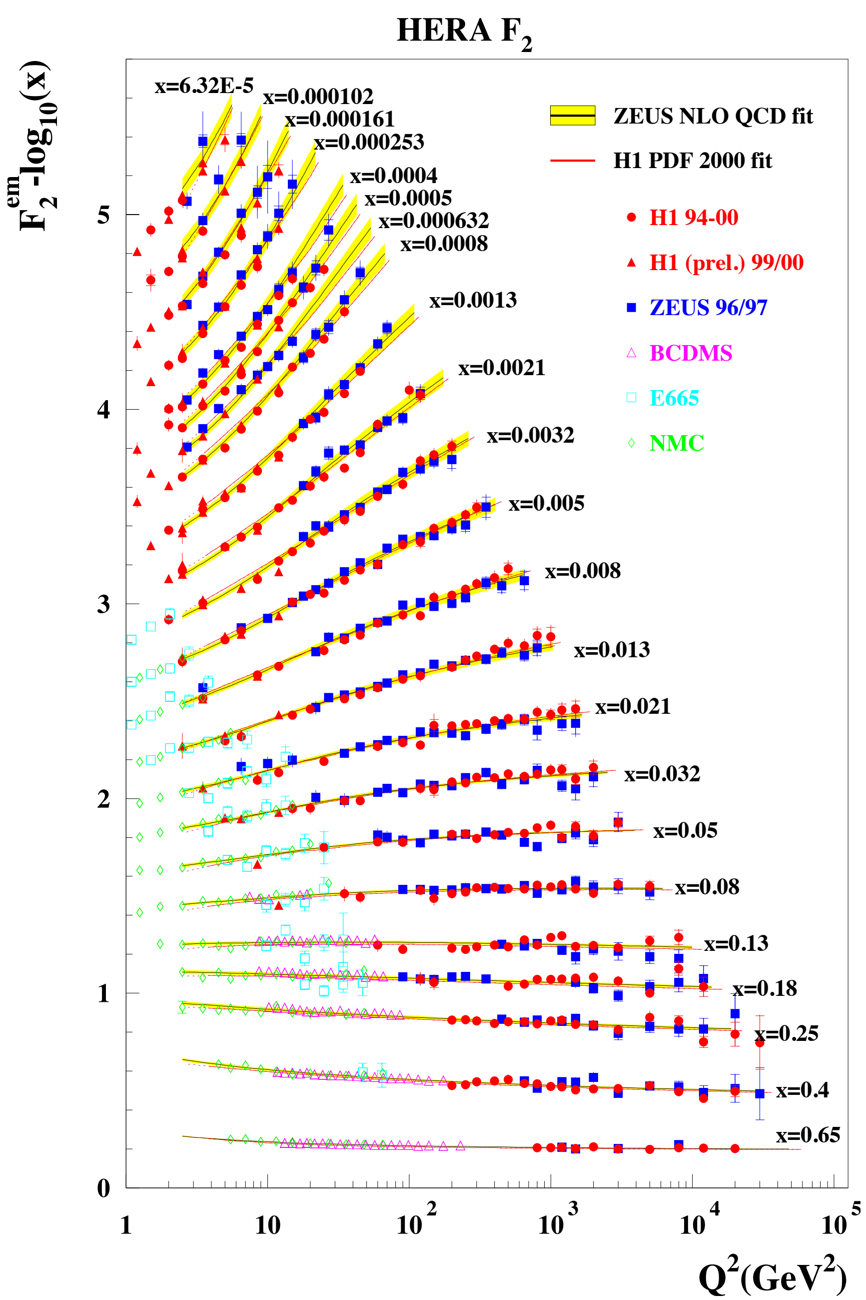
$$W_1(\nu,Q^2) = \sum_i \frac{Q_i^2}{2M} f_i(x) \qquad W_2(\nu,Q^2) = \sum_i \frac{Q_i^2}{\nu} x f_i(x)$$

Leading order result for the form factors

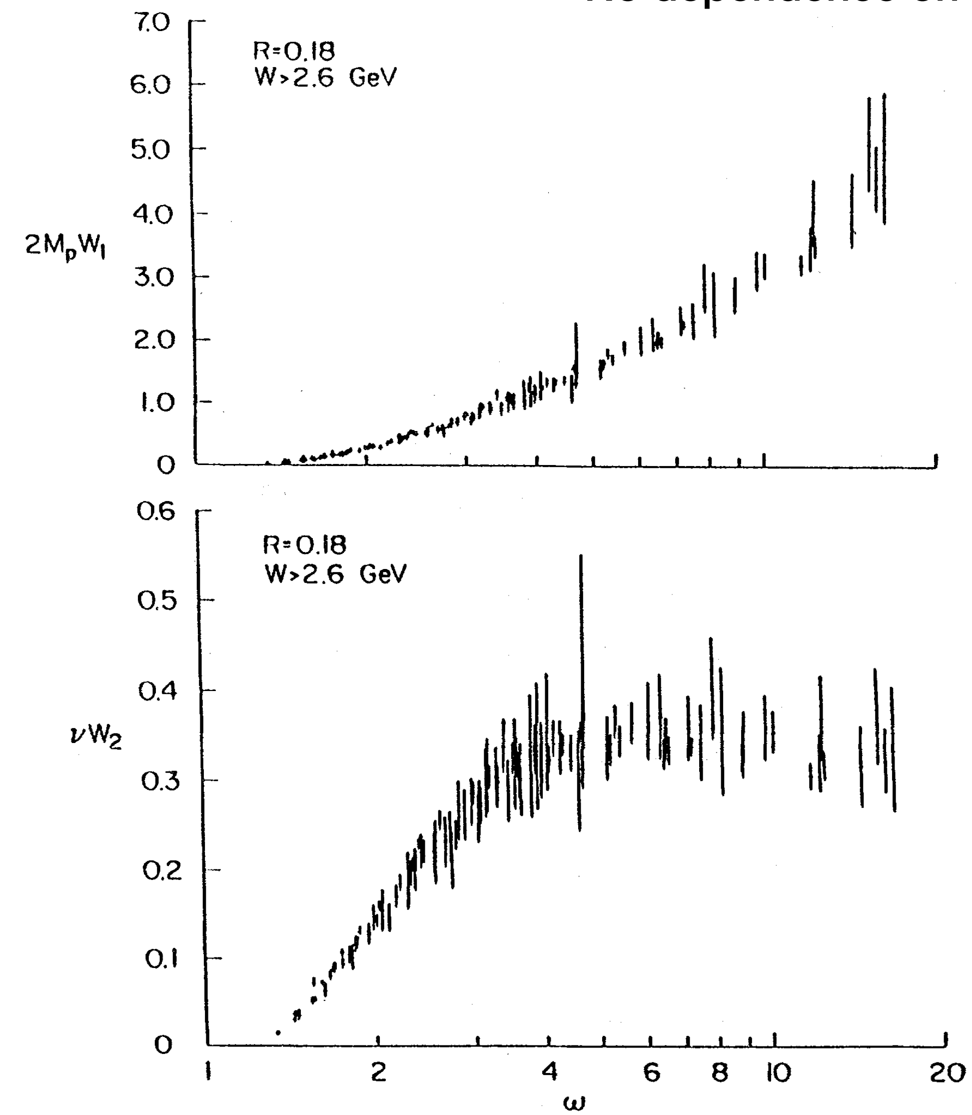
# Bjorken scaling

$$W_1(\nu, Q^2) = \sum_i \frac{Q_i^2}{2M} f_i(x) \qquad W_2(\nu, Q^2) = \sum_i \frac{Q_i^2}{\nu} x f_i(x)$$

$$F_1(x, Q^2) = \sum_i Q_i^2 f_i(x) \quad F_2(x, Q^2) = \sum_i Q_i^2 x f_i(x)$$

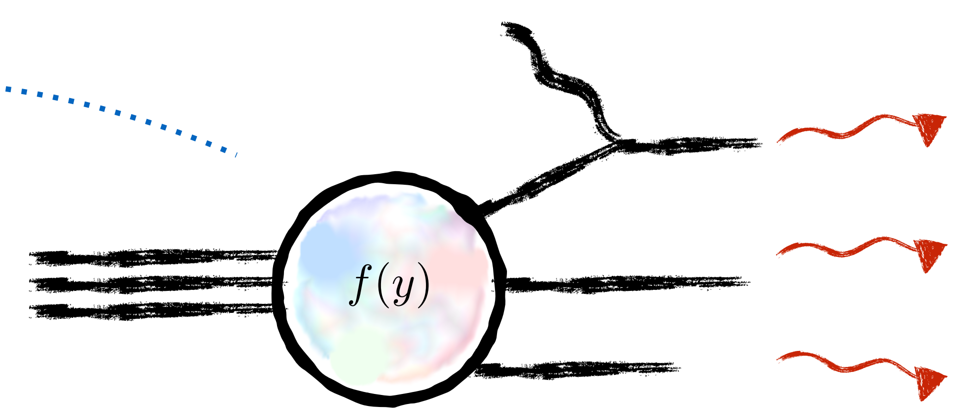



No dependence on Q

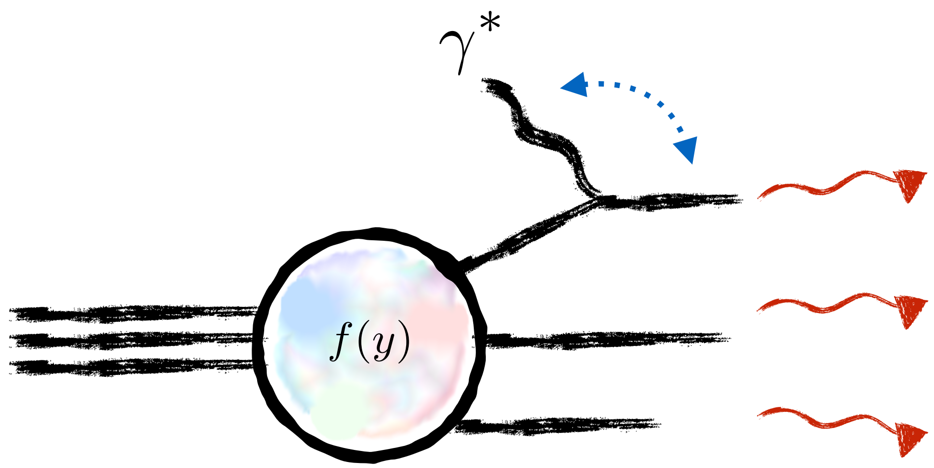


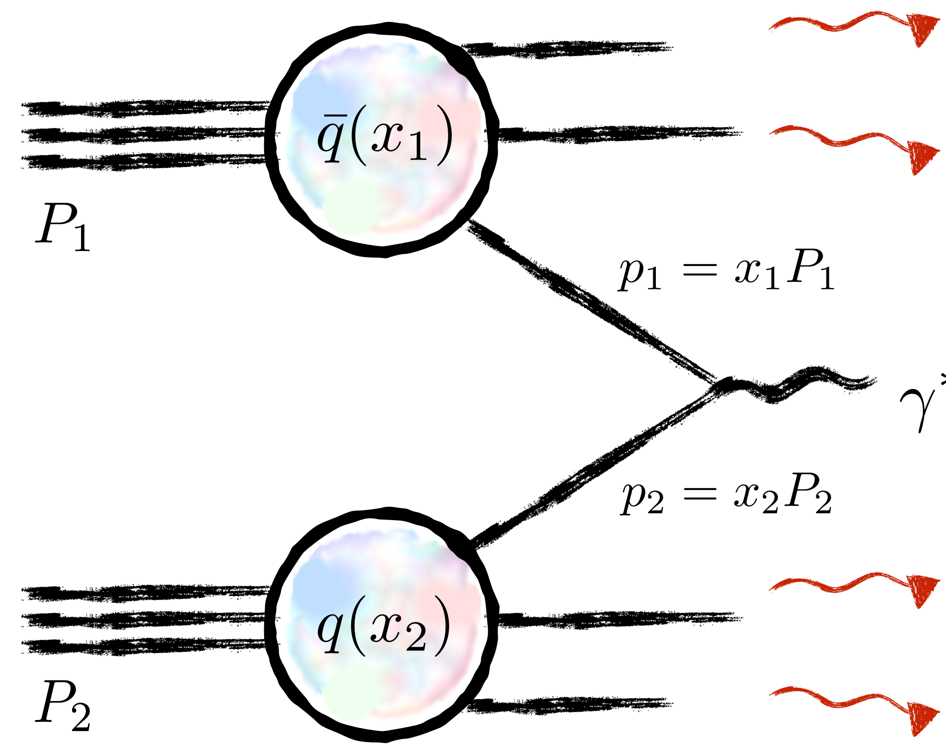

Callan-Gross relation $F_2 = xF_1$

Dominant contribution at large x



# Drell-Yan process




$$s = (p_1 + p_2)^2 = x_1 x_2 S \qquad \tau \equiv s/S \qquad s_0 \equiv M^2$$

"Invariant mass" of the lepton pair

**DIS in the parton model**

$$d\sigma = \int_x^1 dy f(y) d\tilde{\sigma}$$

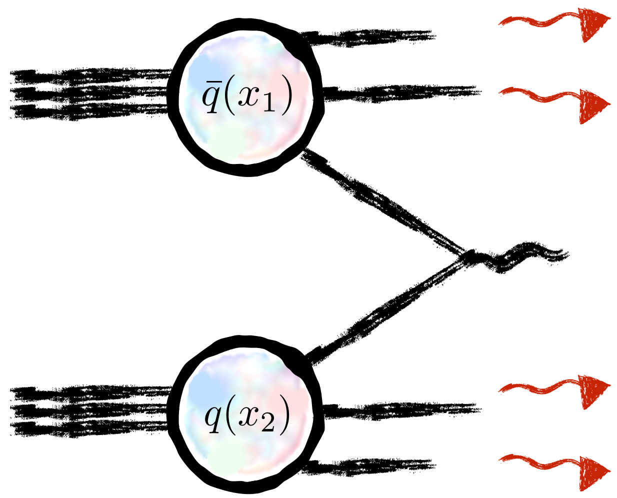
Hadronic subprocess

**Drell-Yan in the parton model**

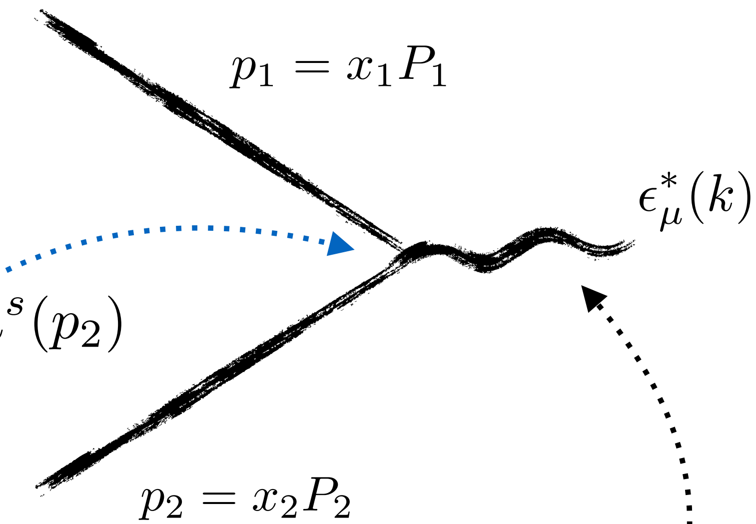
$$d\sigma = \int_{\tau_0}^1 dx_1 \int_{\tau_0/x_1}^1 dx_2 \bar{q}(x_1) q(x_2) d\tilde{\sigma}(q\bar{q} \to \gamma^*)$$

To get the full formula sum over flavors and add another combination of quark-antiquark

# Amplitude of the subprocess




Hadronic subprocess




$$-ig\bar{v}^{s'}(p_1)\gamma^\mu u^s(p_2)$$

Non-perturbative part

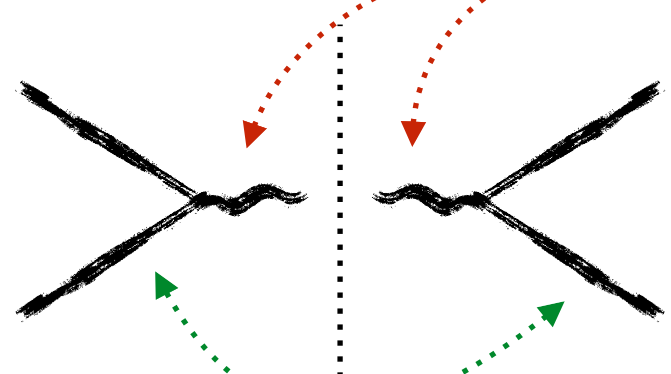
We assume that the photon can be detected directly

In the leading order the result is very simple:

Sum over polarizations

$$\sum_{\text{polarization}} \epsilon^*_\mu(k)\epsilon_\nu(k) \to -g_{\mu\nu}$$

$$iM = -ig\bar{v}^{s'}(p_1)\gamma^\mu u^s(p_2)\epsilon^*_\mu(k)$$

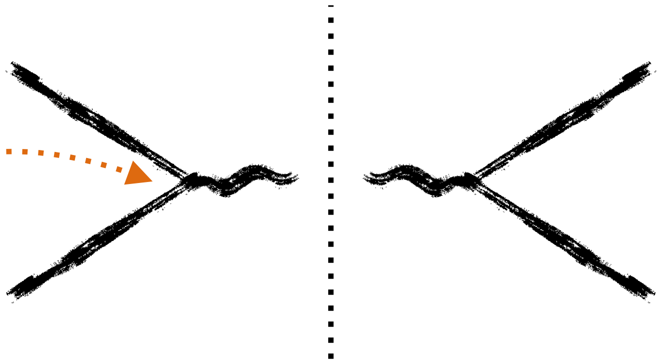


$$-iM^* = ig\bar{u}^s(p_2)\gamma^\nu v^{s'}(p_1)\epsilon_\nu(k)$$

$$\sum_s u^s(p)\bar{u}^s(p) = (\not{p} + m)$$

# Amplitude of the subprocess

$\delta_{il}$



$$\frac{1}{4}\sum_{\text{spin}}|M|^2 = -g_{\mu\nu}\frac{g^2}{4}Tr\{\not p_1\gamma^\mu\not p_2\gamma^\nu\}$$

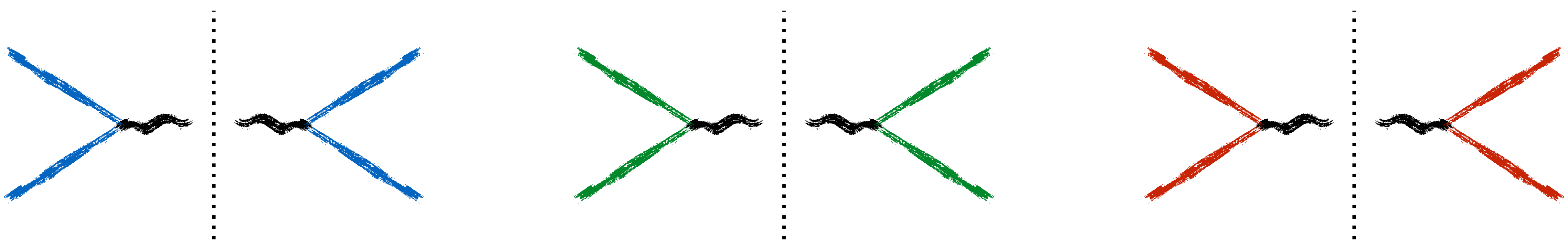
That's not all. We have color!

Average over spin of two quarks

$$\sum_s u_i^s(p)\bar{u}_j^s(p) = (\not p + m)\delta_{ij}$$

$\frac{1}{3}$

$\frac{1}{3}$

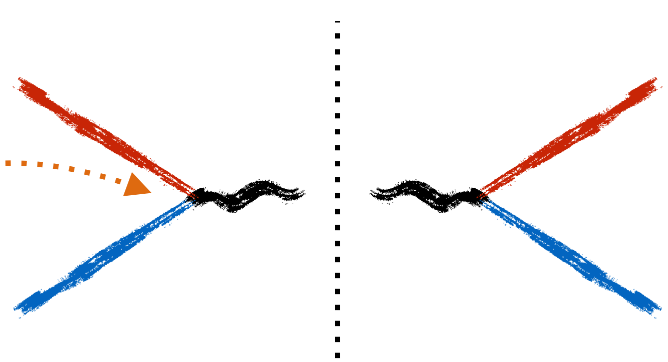


The amplitude doesn't depend on color, but more complex combinations are possible

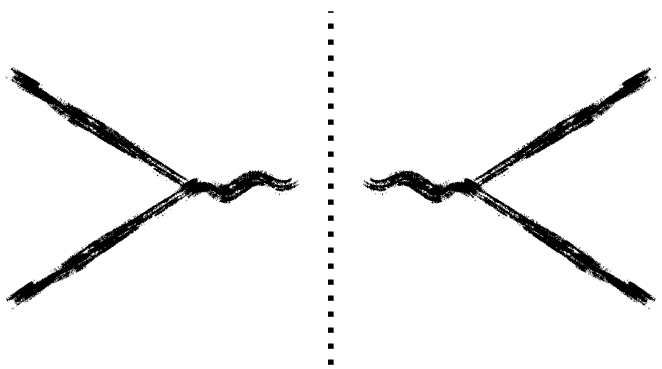
$$\frac{1}{9}\frac{1}{4}\sum_{\text{spin, color}}|M|^2 = -g_{\mu\nu}\frac{1}{3}\frac{g^2}{4}Tr\{\not p_1\gamma^\mu\not p_2\gamma^\nu\}$$

We didn't have this factor in DIS. Why?

$t^a_{il}$



# Amplitude of the subprocess



$$\frac{1}{9}\frac{1}{4}\sum_{\text{spin, color}}|M|^2 = -g_{\mu\nu}\frac{1}{3}\frac{g^2}{4}Tr\{\not p_1\gamma^\mu \not p_2\gamma^\nu\}$$

$$Tr\{\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\} = 4\big(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho}\big)$$

$$\frac{1}{9}\frac{1}{4}\sum_{\text{spin, color}}|M|^2 = \frac{1}{3}\frac{g^2}{4}8p_1\cdot p_2 = \frac{g^2}{3}s$$

Now it is straightforward to write the cross section:

$$d\tilde{\sigma} = \frac{1}{2s}\frac{1}{9\cdot 4}\sum_{\text{spin,color}}|M|^2\frac{d^3k}{(2\pi)^3 2E}(2\pi)^4\delta^4(p_1+p_2-k)$$

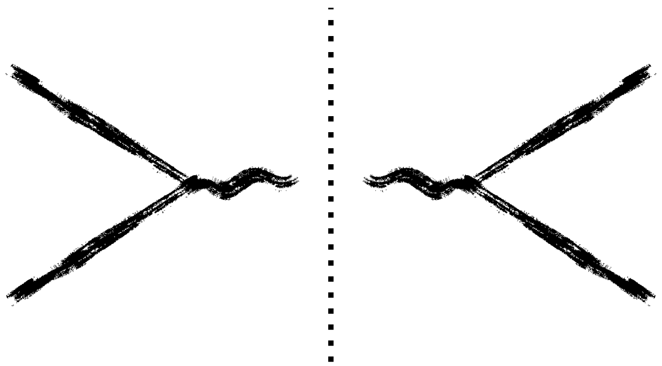
Integral over phase space of the final particle

$$\int\frac{d^3k}{(2\pi)^3 2E}\times(2\pi)^4\delta^4(p_1+p_2-k) = \int\frac{d^4k}{(2\pi)^4}2\pi\delta(k^2-M^2)\times(2\pi)^4\delta^4(p_1+p_2-k)$$

Final result for the partonic cross section

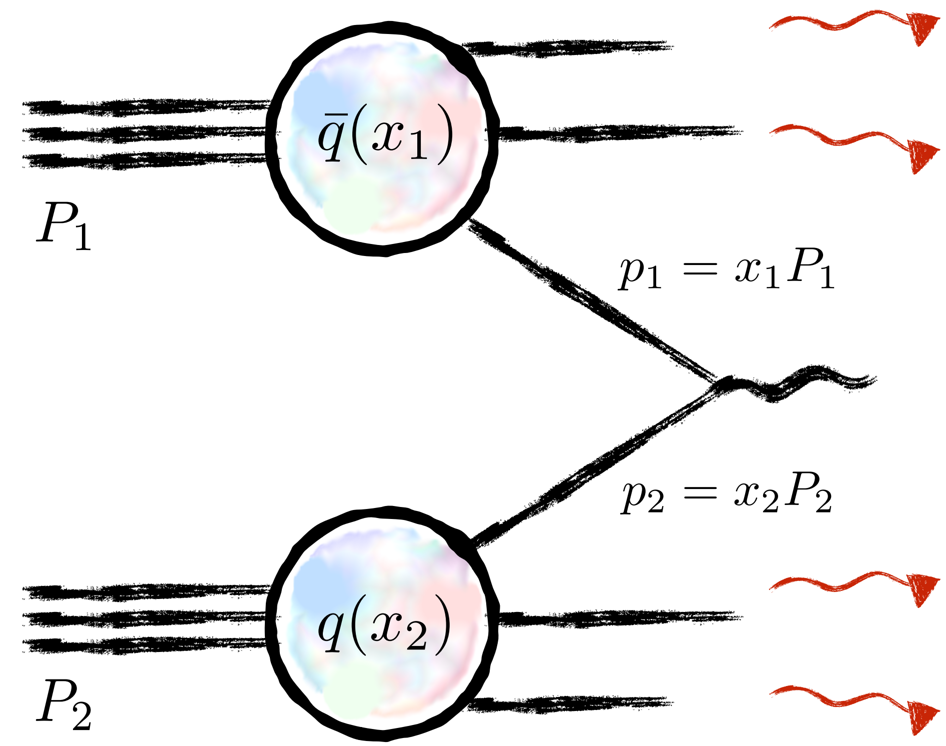
$$d\tilde{\sigma} = \frac{g^2}{6}\times 2\pi\delta(s-M^2)$$

# Cross section for the Drell-Yan process



$$d\tilde{\sigma} = \frac{g^2}{6} \times 2\pi\delta(s - M^2)$$

Substitute




$$d\sigma = \int_{\tau_0}^{1} dx_1 \int_{\tau_0/x_1}^{1} dx_2 \bar{q}(x_1) q(x_2) d\tilde{\sigma}(q\bar{q} \to \gamma^*)$$

$$d\sigma = \frac{g^2}{6} \int_{\tau_0}^{1} dx_1 \int_{\tau_0/x_1}^{1} dx_2 \bar{q}(x_1) q(x_2) \times 2\pi\delta(x_1 x_2 S - M^2)$$

Final result for the Drell-Yan cross section in the parton model

$$d\sigma = \frac{\pi g^2}{3} \frac{1}{S} \sum_f \int_{\tau_0}^{1} \frac{dx}{x} \left[ \bar{q}_f(x) q_f(\frac{\tau_0}{x}) + q_f(x) \bar{q}_f(\frac{\tau_0}{x}) \right]$$

$$\tau_0 \equiv \frac{M^2}{S}$$

Extract from the experiment